Journal of University of Garmian

https://doi.org/10.24271/garmian.scpas13

On Minimal λ_{gc}-Open Sets

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Abstract

In this paper, we defined λ_{gc}-open set by using s-operation and g-closed set, then by using λ_{gc}-open set, we define λ_{gc}-closed set. In addition we define λ_{gc}-closure of subset A of X ($\lambda_{gc}Cl(A)$) and λ_{gc}-interior of subset A of X ($\lambda_{gc}Int(A)$) by using λ_{gc}-closed set and λ_{gc}-open set respectively. Furthermore we introduce and discuss minimal λ_{gc}-open sets in topological spaces. We establish some basic properties of minimal λ_{gc}-open. We obtain an application of a theory of minimal λ_{gc}-open sets and define a λ_{gc}-locally finite space then we prove, Let X be a λ_{gc}-locally finite space and B a nonempty λ_{gc}-open set. Then there exists at least one (finite) minimal λ_{gc}-open set A such that $A \subseteq B$, where λ is semi-regular.

1. Introduction

The study of semi open sets in topological spaces was initiated by Levine[**1**]. The complement of A is denoted by $X \backslash A$. The concept of operation γ was initiated by Kasahara [**2**]. He also introduced γ -closed graph of a function. Using this operation, Ogata[**3**] introduced the concept of γ -open sets and investigated the related topological properties of the associated topology τ_γ and τ. He further investigated general operator approaches of closed graph of mappings. Further Ahmad and Hussain[**4**] continued studying the properties of γ –open(γ-closed) sets. In 2009, Hussain and Ahmad [**5**], introduced the concept of minimal γ-open sets. In 2011[**6**] (resp., in 2013[**7**]) Khalaf and Namiq defined an operation λ called s-operation. They defined λ*-open sets [**8**] which is equivalent to λ-open set[**6**] and λ_s-open set[**7**] by using s-operation. They worked in operation in topology in [9-18].

In this paper, we introduce and discuss minimal λ_{gc}-open sets in topological spaces. We establish some basic properties of minimal λ_{gc}-open sets.
First, we recall some definitions and results used in this paper.

2. Preliminaries

Throughout, X denotes a topological space. Let A be a subset of X, then the closure and the interior of A are denoted by $Cl(A)$ and $Int\,(A)$ respectively. A subset A of a topological space (X,τ) is said to be semi open [**1**] if $A \subseteq Cl\big(Int(A)\big)$ which is defined by Norman Levine . The complement of a semi open set is said to be semi closed [**1**]. The family of all semi open (resp. semi closed) sets in a topological space (X,τ) is denoted by $SO\,(X,\tau)$ or $SO(X)$ (resp. $SC\,(X,\tau)$ or $SC(X)$). And also generalized closed (g-closed) set is defined by him which a subset A of X, $Cl(A) \subseteq G$, when ever $A \subseteq G$, for all open subset G of X [**19**].

Definition 2.1.[7]

We consider λ as a function defined on $SO(X)$ into $P(X)$ and $\lambda: SO(X) \longrightarrow P(X)$ is called an s-operation if $V \subseteq \lambda(V)$ for each non-empty semi open set V.

Remark 2.2. [7]

It is assumed that $\lambda(\phi) = \phi$ and $\lambda(X) = X$ for any s-operation λ.

Definition 2.3.[7]

Let X be a topological space and $\lambda: SO(X) \longrightarrow P(X)$ be an s-operation, then a subset A of X is called a λ^*-open set [**8**]which is equivalent to λ –open set[**6**] and λ_s-open set [**7**] if for each $x \in A$ there exists a semi open set U such that $x \in U$and $\lambda(U) \subseteq A$. The complement of a λ^*-open set is said to be λ^*-closed. The family of all λ^*-open (resp., λ^*-closed) subsets of a topological space $(X, \tau)$is denoted by $SO_\lambda(X, \tau)$ or $SO_\lambda(X)$ (resp., $SC_\lambda(X, \tau)$ or $SC_\lambda(X)$).

Definition 2.4.[7]

A s-operation λ on X is said to be semi-regular which is equivalent to λ -regular [**6**] if for every semi open sets U and V of $x \in X$, there exists a semi open set Wcontaining x such that $\lambda(W) \subseteq \lambda(U) \cap \lambda(V)$.

3. Minimal λ_{gc}-Open Sets

Definition 3.1. Let X be a topological space

1) A λ^*-open[**8**](λ -open[**6**], λ_s-open[**7**]) subset A of X is called λ_{gc}-open if for each $x \in A$ there exists a g-closed set F such that $x \in F \subseteq A$. The complement of a λ_{gc}-open set is called λ_{gc}-closed. The family of all λ_{gc}-open (resp., λ_{gc}-closed) subsets of a topological space (X, τ) is denoted by $SO_{\lambda_{gc}}(X, \tau)$ or $SO_{\lambda_{gc}}(X)$(resp. $SC_{\lambda_{gc}}(X, \tau)$ or $SC_{\lambda_{gc}}(X)$).
2) The λ_{gc}-closure of subset A of X ($\lambda_{gc}Cl(A)$) is the intersection of all λ_{gc}-closed sets containing A.
3) The λ_{gc}-interior of subset A of X ($\lambda_{gc}Int(A)$) is the union of all λ_{gc}-open sets of X contained in A.
4) Let A be a λ_{gc}-open set. Then A is called a minimal λ_{gc}-open set if ϕ and A are the only λ_{gc}-open subsets of A.

Example 3.2

Let $X = \{a, b, c\}$, and $\tau = P(X)$. We define an s-operation $\lambda: SO(X) \longrightarrow P(X)$ as $\lambda(A) = A$ if $A = \{a, c\}$ and $\lambda(A) = X$ otherwise. The λ_{gc}-open sets are ϕ,$\{a, c\}$and X. We have $\{a, c\}$ is minimal λ_{gc}-open set.

Proposition 3.3

Let A be a nonempty λ_{gc}-open subset of a space X. If $A \subseteq \lambda_{gc}Cl(C)$, then $\lambda_{gc}Cl(A) = \lambda_{gc}Cl(C)$, for any nonempty subset C of A.

Proof. For any nonempty subset C of A,we have $\lambda_{gc}Cl(C) \subseteq \lambda_{gc}Cl(A)$. On the other hand, by supposition we see $\lambda_{gc}Cl(A) = \lambda_{gc}Cl(\lambda_{gc}Cl(C)) = \lambda_{gc}Cl(C)$ implies $\lambda_{gc}Cl(A) \subseteq \lambda_{gc}Cl(C)$. Therefore we have $\lambda_{gc}Cl(A) = \lambda_{gc}Cl(C)$ for any nonempty subset $C$of A.

Proposition 3.4

Let A be a nonempty λ_{gc}-open subset of a space X. If $\lambda_{gc}Cl(A) = \lambda_{gc}Cl(C)$, for any nonempty subset C of A, then A is a minimal λ_{gc}-open set.

Proof. Suppose that A is not a minimal λ_{gc}-open set. Then there exists a nonempty λ_{gc}-open set B such that $B \subseteq A$ and hence there exists an element $x \in A$ such that $x \notin B$.Then we have $\lambda_{gc}Cl(\{x\}) \subseteq X \setminus B$ and $\lambda_{gc}Cl(\{x\}) = \lambda_{gc}Cl(A)$. This contradiction proves the proposition.

Remark 3.5

In the remainder of this section we suppose that λ is an s–regular operation defined on a topological space X.

Proposition 3.6

The following statements are true:

(1) If A is a minimal λ_{gc}-open set and B a λ_{gc}-open set. Then $A \cap B = \phi$ or $A \subseteq B$.

(2) If B and C are minimal λ_{gc}-open sets. Then $B \cap C = \phi$ or $B = C$.

Proof.(1) Let B be a λ_{gc}-open set such that $A \cap B \neq \phi$. Since A is a minimal λ_{gc}-open set and $A \cap B \subseteq A$, we have $A \cap B = A$. Therefore $A \subseteq B$.

(2) If $A \cap B \neq \phi$, then by (1), we have $B \subseteq C$ and $C \subseteq B$. Therefore, $B = C$.

Proposition 3.7

Let A be a minimal λ_{gc}-open set. If x is an element of A, then $A \subseteq B$ for any λ_{gc}-open neighborhood B of x.

Proof. Let B be a λ_{gc}-open neighborhood of x such that $A \not\subset B$. Since where λ is λ–regular operation, then $A \cap B$ is λ_{gc}-open set such that $A \cap B \subseteq A$ and $A \cap B \neq \phi$. This contradicts our assumption that A is a minimal λ_{gc}-open set.

Proposition 3.8

Let A be a minimal λ_{gc}-open set. Then for any element x of $A, A = \cap\{B: B$ is λ_{gc}-open neighborhood of $x\}$.

Proof. By Proposition 3.4, and the fact that A is λ_{gc}-open neighborhood of x, we have $A \subseteq \cap\{B: B$ is λ_{gc}-open neighborhood of $x\} \subseteq A$. Therefore, the result follows.

Proposition 3.9

If A is a minimal λ_{gc}-open set in X not containing $x \in X$. Then for any λ_{gc}-open neighborhood C of x, either $C \cap A = \phi$ or $A \subseteq C$.

Proof. Since C is a λ_{gc}-open set, we have the result by Proposition 3.3.

Corollary 3.10

If A is a minimal λ_{gc}-open set in X not containing $x \in X$ such that $x \notin A$. If $A_x = \cap\{B: B$is λ_{gc}-open neighborhood of $x\}$. Then either $A_x \cap A = \phi$ or $A \subseteq A_x$.

Proof. If $A \subseteq B$ for any λ_{gc}-open neighborhood B of x, then $A \subseteq \cap\{B: B$ is λ_{gc}-open neighborhood of $x\}$. Therefore $A \subseteq A_{\mathrm{x}}$. Otherwise there exists a λ_{gc}-open neighborhood B of x such that $B \cap A = \phi$. Then we have $A_{\mathrm{x}} \cap A = \phi$.

Corollary 3.11

If A is a nonempty minimal λ_{gc}-open set of X, then for a nonempty subset $C$of $A, A \subseteq \lambda_{gc}Cl(C)$.

Proof. Let C be any nonempty subset of A. Let $y \in A$ and B be any λ_{gc}-open neighborhood of y. By Proposition 3.4, we have $A \subseteq B$ and $C = A \cap C \subseteq B \cap C$. Thus we have $B \cap C \neq \phi$ and hence $y \in \lambda_{gc}Cl(C)$. This implies that $A \cap \lambda_{gc}Cl(C)$. This completes the proof.

Combining Corollary 3.11 and Propositions 3.3 and 3.4, we have:

Theorem 3.12

Let A be a nonempty λ_{gc}-open subset of space X. Then the following are equivalent:

(1) A is minimal λ_{gc}-open set, where λ is s-regular.

(2) For any nonempty subset C of $A, A \subseteq \lambda_{gc}Cl(C)$.

(3) For any nonempty subset C of A, $\lambda_{gc}Cl(A) = \lambda_{gc}Cl(C)$.

4. Finite λ_{gc}-Open Sets

In this section, we study some properties of minimal λ_{gc}-open sets in finite λ_{gc}-open sets and λ_{gc}-locally finite spaces.

Proposition 4.1

Let (X, τ) be a topological space and $\phi \neq B$ a finite λ_{gc}-open set in X. Then there exists at least one (finite) minimal λ_{gc}-open set A such that $A \subseteq B$.

Proof. Suppose that B is a finite λ_{gc}-open set in X. Then we have the following two possibilities:

(1) B is a minimal λ_{gc}-open set.

(2) B is not a minimal λ_{gc}-open set.

In case (1), if we choose $B = A$, then the proposition is proved. If the case (2) is true, then there exists a nonempty (finite) λ_{gc}-open set B_1which is properly contained in B. If B_1 is minimal λ_{gc}-open, we take $A = B_1$. If B_1 is not a minimal λ_{gc}-open set, then there exists a nonempty (finite) λ_{gc}-open set B_2 such that $B_2 \subseteq B_1 \subseteq B$. We continue this process and have a sequence of λ_{gc}-open sets... $\subseteq B_{\mathrm{m}} \subseteq \cdots \subseteq B_2 \subseteq B_1 \subseteq B$. Since B is a finite, this process will end in a finite number of steps. That is, for some natural number k, we have a minimal λ_{gc}-open set B_k such that $B_k = A$. This completes the proof.

Definition 4.2

A space X is said to be a λ_{gc}-locally finite space, if for each $x \in X$ there exists a finite λ_{gc}-open set A in X such that $x \in A$.

Corollary 4.3

Let X be a λ_{gc}-locally finite space and B a nonempty λ_{gc}-open set. Then there exists at least one (finite) minimal λ_{gc}-open set A such that $A \subseteq B$, where λ is semi-regular.

Proof. Since B is a nonempty set, there exists an element x of B. Since X is a λ_{gc}-locally finite space, we have a finite λ_{gc}-open set B_x such that $x \in B_x$. Since $B \cap B_x$ is a finite λ_{gc}-open set, we get a minimal λ_{gc}-open set A such that $A \subseteq B \cap B_x \subseteq B$ by Proposition 4.1.

Proposition 4.4

Let X be a space and for any $\alpha \in I, B_\alpha$ a λ_{gc}-open set and $\phi \neq A$ a finite λ_{gc}-open set. Then $A \cap \left(\bigcap_{\alpha \in I} B_\alpha\right)$ is a finite λ_{gc}-open set, where λ is $semi$-regular.

Proof. We see that there exists an integer n such that $A \cap \left(\bigcap_{\alpha \in I} B_\alpha\right) = A \cap \left(\bigcap_{i=1}^{n} B_{\alpha i}\right)$ and hence we have the result. Using Proposition 4.4, we can prove the following:

Theorem 4.5

Let X be a space and for any $\alpha \in I, B_\alpha$ a λ_{gc}-open set and for any $\beta \in J, B_\beta$ a nonempty finite λ_{gc}-open set. Then $\left(\bigcup_{\beta \in J} B_\beta\right) \cap \left(\bigcap_{\alpha \in I} B_\alpha\right)$ is a λ_{gc}-open set, where λ is semi-regular.

5. More Properties

Let A be a nonempty finite λ_{gc}-open set. It is clear, by Proposition 3.3 and Proposition 4.1, that if λ is $semi$-regular, then there exists a natural number m such that $\{A_1, A_2, \dots, A_m\}$ is the class of all minimal λ_{gc}-open sets in A satisfying the following two conditions:

(1) For any ι, n with $1 \leq \iota, n \leq m$ and $\iota \neq n, A_\iota \cap A_n = \phi$.

(2) If C is a minimal λ_{gc}-open set in A, then there exists ι with $1 \subseteq \iota \subseteq m$ such that $C = A_\iota$.

Theorem 5.1

Let X be a space and $\phi \neq A$ a finite λ_{gc}-open set such that A is not a minimal λ_{gc}-open set. Let $\{A_1, A_2, \dots, A_m\}$ be a class of all minimal λ_{gc}-open sets in A and $y \in A \setminus (A_1 \cup A_2 \cup \dots \cup A_m)$. Define $A_y = \bigcap\{B : B$ is λ_{gc}-open neighborhood of $x\}$.Then there exists a natural number $k \in \{1,2,3, \dots, m\}$ such that A_k is contained in $A_{y,}$ where λ is $semi$-regular.

Proof. Suppose on the contrary that for any natural number $k \in \{1,2,3, \dots, m\}, A_k$ is not contained in A_y. By Corollary 3.7, for any minimal λ_{gc}-open set A_k in $A, A_k \cap A_y = \phi$. By Proposition 4.4, $\phi \neq A_y$ is a finite λ_{gc}-open set. Therefore by Proposition 4.1, there exists a minimal λ_{gc}-open set C such that $C \subseteq A_y$. Since $C \subseteq A_y \subseteq A$, we have C is a minimal λ_{gc}-open set in A. By supposition, for any minimal λ_{gc}-open set A_k, we have $A_k \cap C \subseteq A_k \cap A_y = \phi$. Therefore, for any natural number $k \in \{1,2,3, \dots, m\}, C \neq A_k$. This contradicts our assumption. Hence the proof.

Proposition 5.2

Let X be a space and $\phi \neq A$ be a finite λ_{gc}-open set which is not a minimal λ_{gc}-open set. Let $\{A_1, A_2, \dots, A_m\}$ be a class of all minimal λ_{gc}-open sets in A and $y \in A \setminus (A_1 \cup A_2 \cup \dots \cup A_m)$. Then there exists a natural number $k \in \{1,2,3, \dots, m\}$, such that for any λ_{gc}-open neighborhood B_y of y, A_k is contained in B_y, where λ is λ-regular.

Proof. This follows from Theorem 5.1, as $\bigcap\{B : B$ is λ_{gc}-open of $y\} \subseteq B_y$. Hence the proof.

Theorem 5.3

Let X be a space and $\phi \neq A$ be a finite λ_{gc}-open set which is not a minimal λ_{gc}-open set. Let $\{A_1, A_2, \dots, A_m\}$ be the class of all minimal λ_{gc}-open sets in A and $y \in A \setminus (A_1 \cup A_2 \cup \dots \cup A_m)$. Then there exists a natural number $k \in \{1,2,3, \dots, m\}$, such that $y \in \lambda_{gc}Cl(A_k)$. where λ is λ-regular.

Proof. It follows from Proposition 5.2, that there exists a natural number $k \in \{1,2,3, \dots, m\}$ such that $A_k \subseteq B$ for any λ_{gc}-open neighborhood B of y. Therefore $\phi \neq A_k \cap A_k \subseteq A_k \cap B$ implies $y \in \lambda_{gc}Cl(A_k)$. This completes the proof.

Proposition 5.4

Let $\phi \neq A$ be a finite λ_{gc}-open set in a space X and for each $k \in \{1,2,3,\dots,m\}$, A_k is a minimal λ_{gc}-open set in A. If the class $\{A_1, A_2, \dots, A_m\}$ contains all minimal λ_{gc}-open sets in A, then for any $\phi \neq B_k \subseteq A_k$, $A \subseteq \lambda_{gc}Cl(B_1 \cup B_2 \cup B_3 \cup \dots \cup B_m)$, where λ is semi-regular.

Proof. If A is a minimal λ_{gc}-open set, then this is the result of Theorem 3.11 (2). Otherwise, when A is not a minimal λ_{gc}-open set. If x is any element of $A \setminus (A_1 \cup A_2 \cup \dots \cup A_m)$, then by Theorem 5.3, $x \in \lambda_{gc}Cl(A_1) \cup \lambda_{gc}Cl(A_2) \cup \dots . \cup \lambda_{gc}Cl(A_m)$. Therefore, by Theorem 3.11(3), we obtain that $A \subseteq \lambda_{gc}Cl(A_1) \cup \lambda_{gc}Cl(A_2) \cup \dots . \cup \lambda_{gc}Cl(A_m) = \lambda_{gc}Cl(B_1) \cup \lambda_{gc}Cl(B_2) \cup \dots . \cup \lambda_{gc}Cl(B_m) = \lambda_{gc}Cl(B_1 \cup B_2 \cup B_3 \cup \dots \cup B_m)$.

Proposition 5.5

Let $\phi \neq A$ be a finite λ_{gc}-open set and A_k is a minimal λ_{gc}-open set in A, for each $k \in \{1,2,3,\dots,m\}$. If for any $\phi \neq B_k \subseteq A_k$, $A \subseteq \lambda_{gc}Cl(B_1 \cup B_2 \cup B_3 \cup \dots \cup B_m)$ then $\lambda_{gc}Cl(A) = \lambda_{gc}Cl(B_1 \cup B_2 \cup B_3 \cup \dots \cup B_m)$.

Proof. For any $\phi \neq B_k \subseteq A_k$ with $k \in \{1,2,3,\dots,m\}$, we have $\lambda_{gc}Cl(B_1 \cup B_2 \cup B_3 \cup \dots \cup B_m) \subseteq \lambda_{gc}Cl(A)$. Also, we have $\lambda_{gc}Cl(A) \subseteq \lambda_{gc}Cl(B_1) \cup \lambda_{gc}Cl(B_2) \cup \dots . \cup \lambda_{gc}Cl(B_m) = \lambda_{gc}Cl(B_1 \cup B_2 \cup B_3 \cup \dots \cup B_m)$. Therefore, $\lambda_{gc}Cl(A) = \lambda_{gc}Cl(B_1 \cup B_2 \cup B_3 \cup \dots \cup B_m)$ for any nonempty subset B_k of A_k with $k \in \{1,2,3,\dots,m\}$.

Proposition 5.6

Let $\phi \neq A$ be a finite λ_{gc}-open set and for each $k \in \{1,2,3,\dots,m\}$, A_k is a minimal λ_{gc}-open set in A. If for any $\phi \neq B_k \subseteq A_k$, $\lambda_{gc}Cl(A) = \lambda_{gc}Cl(B_1 \cup B_2 \cup B_3 \cup \dots \cup B_m)$, then the class $\{A_1, A_2, \dots, A_m\}$ contains all minimal λ_{gc}-open sets in A.

Proof. Suppose that C is a minimal λ_{gc}-open set in A and $C \neq A_k$ for $k \in \{1,2,3,\dots,m\}$. Then we have $C \cap \lambda_{gc}Cl(A_k) = \phi$ for each $k \in \{1,2,3,\dots,m\}$. It follows that any element of C is not contained in $\lambda_{gc}Cl(A_1 \cup A_2 \cup \dots \cup A_m)$. This is a contradiction to the fact that $C \subseteq A \subseteq \lambda_{gc}Cl(A) = \lambda_{gc}Cl(B_1 \cup B_2 \cup B_3 \cup \dots \cup B_m)$. This completes the proof.

Combining Propositions 5.4, 5.5 and 5.6, we have the following theorem:

Theorem 5.7

Let A be a nonempty finite λ_{gc}-open set and A_k a minimal λ_{gc}-open set in A for each $k \in \{1,2,3,\dots,m\}$. Then the following three conditions are equivalent:

(1) The class $\{A_1, A_2, \dots, A_m\}$ contains all minimal λ_{gc}-open sets in A.

(2) For any $\phi \neq B_k \subseteq A_k$, $A \subseteq \lambda_{gc}Cl(B_1 \cup B_2 \cup B_3 \cup \dots \cup B_m)$.

(3) For any $\phi \neq B_k \subseteq A_k$, $\lambda_{gc}Cl(A) = \lambda_{gc}Cl(B_1 \cup B_2 \cup B_3 \cup \dots \cup B_m)$, where λ is semi-regular.

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