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On Minimal $\lambda_{\alpha c}$-Open Sets

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Abstract

In this paper, we introduce and discuss minimal $\lambda_{\alpha c}$-open sets in topological spaces. We establish some basic properties of minimal $\lambda_{\alpha c}$-open. We obtain an application of a theory of minimal $\lambda_{\alpha c}$-open sets and we defined a $\lambda_{\alpha c}$-locally finite space.

1. Introduction

The study of semi open sets in topological spaces was initiated by Levine[1]. The complement of A is denoted by $X\backslash A$. In the space (X,τ), a subset A is said to be α-open [2] if $A \subseteq Int(Cl(Int(A)))$. The family of all b-open sets of (X,τ) is denoted by $BO(X)$. The complement of α-open is called α-closed. The concept of operation γ was initiated by Kasahara [3]. He also introduced γ -closed graph of a function. Using this operation, Ogata[4] introduced the concept of γ -open sets and investigated the related topological properties of the associated topology τ_γ and τ. He further investigated general operator approaches of closed graph of mappings. Further Ahmad and Hussain[5] continued studying the properties of γ –open(γ-closed) sets. In 2009, Hussainand Ahmad [6], introduced the concept of minimal γ-open sets. In 2011[7] (resp., in 2013[8]) Khalaf and Namiq defined an operation λ called s-operation. They defined λ*-open sets [9] which is equivalent to λ-open set[7] and λ_s- open set[8] by using s-operation. They work in operation in topology in [10-22]. They defined $\lambda_{\beta c}$-open set by using s-operation and β-closed set and also investigated several properties of $\lambda_{\beta c}$-derived, $\lambda_{\beta c}$-interior and $\lambda_{\beta c}$-closure points in topological spaces.

In this paper, we introduce and discuss minimal $\lambda_{\alpha c}$-open sets in topological spaces.

We establish some basic properties of minimal $\lambda_{\alpha c}$-open sets and provide an example to illustrate that minimal $\lambda_{\alpha c}$-open sets are independent of minimal open sets.

First, we recall some definitions and results used in this paper.

2. Preliminaries

Throughout, X denotes a topological space. Let A be a subset of X, then the closure and the interior of A are denoted by $Cl(A)$ and $Int(A)$ respectively. A subset A of a topological space (X,τ) is said to be semi open [1] if $A \subseteq Cl\big(Int(A)\big)$. The complement

of a semi open set is said to be semi closed [1]. The family of all semi open (resp. semi closed) sets in a topological space (X, τ) is denoted by $SO(X, \tau)$ or $SO(X)$ (resp. $SC(X, \tau)$ or $SC(X)$). We consider λ as a function defined on $SO(X)$ into $P(X)$ and $\lambda: SO(X) \longrightarrow P(X)$ is called an s-operation if $V \subseteq \lambda(V)$ for each non-empty semi open set V. It is assumed that $\lambda(\phi) = \phi$ and $\lambda(X) = X$ for any s-operation λ. Let X be a topological space and $\lambda: SO(X) \longrightarrow P(X)$ be an s-operation, then a subset A of X is called a λ^*-open set [9]which is equivalent to λ –open set[7] and λ_s-open set [8] if for each $x \in A$ there exists a semi open set U such that $x \in U$and $\lambda(U) \subseteq A$.
The complement of a λ^*-open set is said to be λ^*-closed. The family of all λ^*-open (resp., λ^*-closed) subsets of a topological space $(X, \tau)$is denoted by $SO_\lambda(X, \tau)$ or $SO_\lambda(X)$ (resp. , $SC_\lambda(X, \tau)$ or $SC_\lambda(X)$).

Definition 2.1.A λ^*-open[9](λ -open[7], λ_s-open[8]) subset A of a topological space X is called $\lambda_{\beta c}$-open [23] if for each $x \in A$ there exists a β-closed set F such that $x \in F \subseteq A$. The complement of a $\lambda_{\beta c}$-open set is called $\lambda_{\beta c}$-closed[23]. The family of all $\lambda_{\beta c}$-open (resp., $\lambda_{\beta c}$-closed) subsets of a topological space (X, τ) is denoted by $SO_{\lambda_{\beta c}}(X, \tau)$ or $SO_{\lambda_{\beta c}}(X)$(resp. $SC_{\lambda_{\beta c}}(X, \tau)$or $SC_{\lambda_{\beta c}}(X)$) [23].

We get the following results in [23]

Proposition 2.2. For a topological space X, $SO_{\lambda_{\beta c}}(X) \subseteq SO_\lambda(X) \subseteq SO(X)$.

The following example shows that the converse of the above proposition may not be true in general.

Example 2.3. Let $X = \{a, b, c\}$, and $\tau = \{\phi, \{a\}, X\}$. We define an s-operation $\lambda: SO(X) \longrightarrow P(X)$ as $\lambda(A) = A$ if $b \in A$ and $\lambda(A) = X$ otherwise. Here, we have $\{a, c\}$ is semi open but it is not λ^*-open. And also $\{a, b\}$ is λ^*-open set but it is not $\lambda_{\alpha c}$-open .

Definition 2.4. An s-operation λ on X is said to be s-regular which is equivalent to λ -regular [8]if for every semi open sets $U$and V of $x \in X$, there exists a semi open set Wcontaining x such that $\lambda(W) \subseteq \lambda(U) \cap \lambda(V)$.

Definition 2.5. Let A be a subset of X. Then:

(1) The $\lambda_{\beta c}$-closure of A ($\lambda_{\beta c}Cl(A)$) is the intersection of all $\lambda_{\beta c}$-closed sets containing A.

(2) The $\lambda_{\beta c}$-interior of A ($\lambda_{\beta c}Int(A)$) is the union of all $\lambda_{\beta c}$-open sets of X contained in A.

Proposition 2.6. For each point $x \in X, x \in \lambda_{\beta c}Cl(A)$ if and only if $V \cap A \neq \phi$ for every $V \in SO_{\lambda_{\beta c}}(X)$ such that $x \in V$.

Proposition 2.7. Let $\{A_\alpha\}_{\alpha\in I}$be any collection of $\lambda_{\beta c}$-open sets in a topological space (X,τ), then $\bigcup_{\alpha\in I} A_\alpha$is a $\lambda_{\beta c}$-open set.

Proposition 2.8. Let λ bean s-regular s-operation. If A and B are $\lambda_{\beta c}$-open sets in X, then $A\cap B$is also a $\lambda_{\beta c}$-open set.

The proof of the following two propositions are in [24].

Proposition 2.9. Let $\{A_\alpha\}_{\alpha\in I}$ be any collection of λ^*-open sets in a topological space (X,τ), then $\bigcup_{\alpha\in I} A_\alpha$ is a λ^*-open set.

Proposition 2.10. Let λ be semi-regular operation. If A and B are λ^*-open sets in X, then $A\cap B$ is also a λ^*-open set.

Definition 2.11. A λ^*-open[9](λ -open[7], λ_s-open[8]) subset A of a topological space X is called $\lambda_{\alpha c}$-open if for each $x\in A$ there exists a b-closed set F such that $x\in F\subseteq A$. The complement of a $\lambda_{\alpha c}$-open set is called $\lambda_{\alpha c}$-closed. The family of all $\lambda_{\alpha c}$-open (resp., $\lambda_{\alpha c}$-closed) subsets of a topological space(X,τ) is denoted by $SO_{\lambda_{\alpha c}}(X,\tau)$ or $SO_{\lambda_{\alpha c}}(X)$(resp. $SC_{\lambda_{\alpha c}}(X,\tau)$or $SC_{\lambda_{\alpha c}}(X)$).

Proposition 2.12. For a topological space $X, SO_{\lambda_{\alpha c}}(X)\subseteq SO_\lambda(X)\subseteq SO(X)$.

Proof. Obvious.

The following example shows that the converse of the above proposition may not be true in general.

Example 2.13. In Example 2.3**,** we have $\{a,c\}$ is semi open but it is not λ^*-open. And also $\{a,b\}$ is λ^*-open set but it is not $\lambda_{\alpha c}$-open .

Definition 2.14. An s-operation λ on X is said to be s-regular which is equivalent to λ -regular [8] if for every semi open sets U and V of $x\in X$, there exists a semi open set Wcontaining x such that $\lambda(W)\subseteq\lambda(U)\cap\lambda(V)$.

Definition 2.15. Let A be a subset of X. Then:

(3) The $\lambda_{\alpha c}$-closure of A ($\lambda_{\alpha c}Cl(A)$) is the intersection of all $\lambda_{\alpha c}$-closed sets containing A.

(4) The $\lambda_{\alpha c}$-interior of A ($\lambda_{\alpha c}Int(A)$) is the union of all $\lambda_{\alpha c}$-open sets of X contained in A.

Proposition 2.16. For each point $x\in X, x\in\lambda_{\alpha c}Cl(A)$ if and only if $V\cap A\neq\phi$ for every $V\in SO_{\lambda_{\alpha c}}(X)$ such that $x\in V$.

Proof. Obvious

Proposition 2.17. Let $\{A_\alpha\}_{\alpha\in I}$be any collection of $\lambda_{\alpha c}$ -open sets in a topological space (X,τ), then $\bigcup_{\alpha\in I} A_\alpha$is a $\lambda_{\alpha c}$-open set.

Proof. Obvious

Proposition 2.18. Let λ bean s-regular s-operation. If A and B are $\lambda_{\alpha c}$-open sets in X, then $A \cap B$ is also a $\lambda_{\alpha c}$-open set.

Proof. Obvious

3. Minimal $\lambda_{\alpha c}$-Open Sets

Definition 3.1. Let X be a space and $A \subseteq X$ be a $\lambda_{\alpha c}$-open set. Then A is called a minimal $\lambda_{\alpha c}$-open set if ϕ and A are the only $\lambda_{\alpha c}$-open subsets of A.

Example 3.2. Let $X = \{a, b, c\}$, and $\tau = P(X)$. We define an s-operation $\lambda: SO(X) \longrightarrow P(X)$ as $\lambda(A) = A$ if $A = \{a, c\}$ and $\lambda(A) = X$ otherwise. The $\lambda_{\alpha c}$-open sets are $\phi, \{a, c\}$and X. We have $\{a, c\}$ is minimal $\lambda_{\alpha c}$-open set.

Proposition 3.3. Let A be a nonempty $\lambda_{\alpha c}$-open subset of a space X. If $A \subseteq \lambda_{\alpha c}Cl(C)$, then $\lambda_{\alpha c}Cl(A) = \lambda_{\alpha c}Cl(C)$, for any nonempty subset C of A.

Proof. For any nonempty subset C of A,we have $\lambda_{\alpha c}Cl(C) \subseteq \lambda_{\alpha c}Cl(A)$. On the other hand, by supposition we see $\lambda_{\alpha c}Cl(A) = \lambda_{\alpha c}Cl(\lambda_{\alpha c}Cl(C))$=$\lambda_{\alpha c}Cl(C)$ implies $\lambda_{\alpha c}Cl(A) \subseteq \lambda_{\alpha c}Cl(C)$.

Therefore we have $\lambda_{\alpha c}Cl(A) = \lambda_{\alpha c}Cl(C)$ for any nonempty subset $C$of A.

Proposition 3.4. Let A be a nonempty $\lambda_{\alpha c}$-open subset of a space X.If $\lambda_{\alpha c}Cl(A) = \lambda_{\alpha c}Cl(C)$, for any nonempty subset C of A, then A is a minimal $\lambda_{\alpha c}$-open set.

Proof. Suppose that A is not a minimal $\lambda_{\alpha c}$-open set. Then there exists a nonempty $\lambda_{\alpha c}$-open set B such that $B \subseteq A$ and hence there exists an element $x \in A$ such that $x \notin B$.Then we have $\lambda_{\alpha c}Cl(\{x\}) \subseteq X \setminus B$ implies that $\lambda_{\alpha c}Cl(\{x\}) = \lambda_{\alpha c}Cl(A)$.This contradiction proves the proposition.

Remark 3.5. In the remainder of this section we suppose that λ is an s–regular operation defined on a topological space X.

Proposition 3.6. The following statements are true:

(1) If A is a minimal $\lambda_{\alpha c}$-open set and B a $\lambda_{\alpha c}$-open set. Then $A \cap B = \phi$ or $A \subseteq B$.

(2) If B and C are minimal $\lambda_{\alpha c}$-open sets. Then $B \cap C = \phi$ or $B = C$.

Proof.(**1**) Let B be a $\lambda_{\alpha c}$-open set such that $A \cap B \neq \phi$. Since A is a minimal $\lambda_{\alpha c}$-open set and $A \cap B \subseteq A$, we have $A \cap B = A$. Therefore $A \subseteq B$.

(**2**) If $A \cap B \neq \phi$, then by (1), we have $B \subseteq C$ and $C \subseteq B$. Therefore, $B = C$.

Proposition 3.7. Let A be a minimal $\lambda_{\alpha c}$-open set. If x is an element of A, then $A \subseteq B$ for any $\lambda_{\alpha c}$-open neighborhood B of x.

Proof. Let B be a $\lambda_{\alpha c}$-open neighborhood of x such that $A \not\subset B$. Since where λ is λ–regular operation, then $A \cap B$ is $\lambda_{\alpha c}$-open set such that $A \cap B \subseteq A$ and $A \cap B \neq \phi$. This contradicts our assumption that A is a minimal $\lambda_{\alpha c}$-open set.

Proposition 3.8. Let A be a minimal $\lambda_{\alpha c}$-open set. Then for any element x of $A, A = \bigcap\{ B: B$ is $\lambda_{\alpha c}$-open neighborhood of $x \}$.

Proof. By Proposition 3.4, and the fact that A is $\lambda_{\alpha c}$-open neighborhood of x, we have $A \subseteq \bigcap\{ B: B$ is $\lambda_{\alpha c}$-open neighborhood of $x \} \subseteq A$. Therefore, the result follows.

Proposition 3.9. If A is a minimal $\lambda_{\alpha c}$-open set in X not containing $x \in X$. Then for any $\lambda_{\alpha c}$-open neighborhood C of x, either $C \cap A = \phi$ or $A \subseteq C$.

Proof. Since C is a $\lambda_{\alpha c}$-open set, we have the result by Proposition 3.3.

Corollary 3.10. If A is a minimal $\lambda_{\alpha c}$-open set in X not containing $x \in X$ such that $x \notin A$. If $A_x = \bigcap\{ B: B$is $\lambda_{\alpha c}$-open neighborhood of $x \}$. Then either $A_{\mathrm{x}} \cap A = \phi$ or $A \subseteq A_x$.

Proof. If $A \subseteq B$ for any $\lambda_{\alpha c}$-open neighborhood B of x, then $A \subseteq \bigcap\{ B: B$ is $\lambda_{\alpha c}$-open neighborhood of $x \}$. Therefore $A \subseteq A_{\mathrm{x}}$. Otherwise there exists a $\lambda_{\alpha c}$-open neighborhood B of x such that $B \cap A = \phi$. Then we have $A_{\mathrm{x}} \cap A = \phi$.

Corollary 3.11. If A is a nonempty minimal $\lambda_{\alpha c}$-open set of X, then for a nonempty subset $C$of $A, A \subseteq \lambda_{\alpha c}Cl(C)$.

Proof. Let C be any nonempty subset of A. Let $y \in A$ and B be any $\lambda_{\alpha c}$-open neighborhood of y. By Proposition 3.4, we have $A \subseteq B$ and $C = A \cap C \subseteq B \cap C$. Thus we have $B \cap C \neq \phi$ and hence $y \in \lambda_{\alpha c}Cl(C)$. This implies that $A \cap \lambda_{\alpha c}Cl(C)$. This completes the proof.

Combining Corollary 3.11 and Propositions 3.3 and 3.4, we have:

Theorem 3.11. Let A be a nonempty $\lambda_{\alpha c}$-open subset of space X. Then the following are equivalent:

(1) A is minimal $\lambda_{\alpha c}$-open set, where λ is s-regular.

(2) For any nonempty subset C of $A, A \subseteq \lambda_{\alpha c}Cl(C)$.

(3) For any nonempty subset C of A, $\lambda_{\alpha c}Cl(A) = \lambda_{\alpha c}Cl(C)$.

4. Finite $\lambda_{\alpha c}$ -Open Sets

In this section, we study some properties of minimal $\lambda_{\alpha c}$-open sets in finite $\lambda_{\alpha c}$-open sets and $\lambda_{\alpha c}$-locally finite spaces.

Proposition 4.1. Let (X, τ) be a topological space and $\phi \neq B$ a finite $\lambda_{\alpha c}$-open set in X. Then there exists at least one (finite) minimal $\lambda_{\alpha c}$-open set A such that $A \subseteq B$.

Proof. Suppose that B is a finite $\lambda_{\alpha c}$-open set in X. Then we have the following two possibilities:

(1) B is a minimal $\lambda_{\alpha c}$-open set.

(2) B is not a minimal b-open set.

In case (1), if we choose $B = A,$ then the proposition is proved. If the case (2) is true, then there exists a nonempty (finite) $\lambda_{\alpha c}$-open set B_1 which is properly contained in B. If B_1 is minimal $\lambda_{\alpha c}$-open, we take $A = B_1$. If B_1 is not a minimal $\lambda_{\alpha c}$-open set, then there exists a nonempty (finite) $\lambda_{\alpha c}$-open set B_2 such that $B_2 \subseteq B_1 \subseteq B$. We continue this process and have a sequence of $\lambda_{\alpha c}$-open sets... $\subseteq B_{\mathrm{m}} \subseteq \cdots \subseteq B_2 \subseteq B_1 \subseteq B$. Since B is a finite, this process will end in a finite number of steps. That is, for some natural number $k,$ we have a minimal $\lambda_{\alpha c}$-open set B_k such that $B_k = A$. This completes the proof.

Definition 4.2. A space X is said to be a $\lambda_{\alpha c}$-locally finite space, if for each $x \in X$ there exists a finite $\lambda_{\alpha c}$-open set A in X such that $x \in A$.

Corollary 4.3. Let X be a $\lambda_{\alpha c}$-locally finite space and B a nonempty $\lambda_{\alpha c}$-open set. Then there exists at least one (finite) minimal $\lambda_{\alpha c}$-open set A such that $A \subseteq B,$ where λ is $semi$-regular.

Proof. Since B is a nonempty set, there exists an element x of B. Since X is a $\lambda_{\alpha c}$-locally finite space, we have a finite $\lambda_{\alpha c}$-open set B_x such that $x \in B_x$. Since $B \cap B_x$ is a finite $\lambda_{\alpha c}$-open set, we get a minimal $\lambda_{\alpha c}$-open set A such that $A \subseteq B \cap B_x \subseteq B$ by Proposition 4.1.

Proposition 4.4. Let X be a space and for any $\alpha \in I, B_\alpha$ a $\lambda_{\alpha c}$-open set and $\phi \neq A$ a finite $\lambda_{\alpha c}$-open set. Then $A \cap (\bigcap_{\alpha \in I} B_\alpha)$ is a finite $\lambda_{\alpha c}$-open set, where λ is $semi$-regular.

Proof. We see that there exists an integer n such that $A \cap (\bigcap_{\alpha \in I} B_\alpha) = A \cap (\bigcap_{i=1}^{n} B_{\alpha i})$ and hence we have the result.

Using Proposition 4.4, we can prove the following:

Theorem 4.5. Let X be a space and for any $\alpha \in I, B_\alpha$ a $\lambda_{\alpha c}$-open set and for any $\beta \in J, B_\beta$ a nonempty finite $\lambda_{\alpha c}$-open set. Then $(\bigcup_{\beta \in J} B_\beta) \cap (\bigcap_{\alpha \in I} B_\alpha)$ is a $\lambda_{\alpha c}$-open set, where λ is $semi$-regular.

5. More Properties

Let A be a nonempty finite $\lambda_{\alpha c}$-open set. It is clear, by Proposition 3.3 and Proposition 4.1, that if λ is $semi$-regular, then there exists a natural number m such that $\{A_1, A_2, \dots, A_m\}$ is the class of all minimal $\lambda_{\alpha c}$-open sets in A satisfying the following two conditions:

(1) For any ι, n with $1 \leq \iota, n \leq m$ and $\iota \neq n, A_\iota \cap A_n = \phi$.

(2) If C is a minimal $\lambda_{\alpha c}$-open set in $A,$ then there exists ι with $1 \subseteq \iota \subseteq m$ such that $C = A_\iota$.

Theorem 5.1. Let X be a space and $\phi \neq A$ a finite $\lambda_{\alpha c}$-open set such that A is not a minimal $\lambda_{\alpha c}$-open set. Let $\{A_1, A_2, \dots, A_m\}$ be a class of all minimal $\lambda_{\alpha c}$-open sets in A and $y \in A \setminus (A_1 \cup A_2 \cup \dots \cup A_m)$. Define $A_y = \bigcap\{\, B: B$ is $\lambda_{\alpha c}$-open neighborhood of $x\,\}$. Then there exists a natural number $k \in \{1,2,3, \dots, m\}$ such that A_k is contained in $A_{y,}$ where λ is $semi$-regular.

Proof. Suppose on the contrary that for any natural number $k \in \{1,2,3, \dots, m\}, A_k$ is not contained in A_y. By Corollary 3.7, for any minimal $\lambda_{\alpha c}$-open set A_k in $A, A_k \cap A_y = \phi$. By Proposition 4.4, $\phi \neq A_y$ is a finite $\lambda_{\alpha c}$-open set. Therefore by Proposition 4.1, there exists a minimal $\lambda_{\alpha c}$-open set C such that $C \subseteq A_y$. Since $C \subseteq A_y \subseteq A$, we have C is a minimal $\lambda_{\alpha c}$-open set in A. By supposition, for any minimal $\lambda_{\alpha c}$-open set A_k, we have $A_k \cap C \subseteq A_k \cap A_y = \phi$. Therefore, for any natural number $k \in \{1,2,3, \dots, m\}, C \neq A_k$. This contradicts our assumption. Hence the proof.

Proposition 5.2. Let X be a space and $\phi \neq A$ be a finite $\lambda_{\alpha c}$-open set which is not a minimal $\lambda_{\alpha c}$-open set. Let $\{A_1, A_2, \dots, A_m\}$ be a class of all minimal $\lambda_{\alpha c}$-open sets in A and $y \in A \setminus (A_1 \cup A_2 \cup \dots \cup A_m)$. Then there exists a natural number $k \in \{1,2,3, \dots, m\}$, such that for any $\lambda_{\alpha c}$-open neighborhood B_y of y, A_k is contained in B_y, where λ is λ-regular.

Proof. This follows from Theorem 5.1, as $\bigcap\{\, B: B$ is $\lambda_{\alpha c}$-open of $y\,\} \subseteq B_y$. Hence the proof.

Theorem 5.3. Let X be a space and $\phi \neq A$ be a finite $\lambda_{\alpha c}$-open set which is not a minimal $\lambda_{\alpha c}$-open set. Let $\{A_1, A_2, \dots, A_m\}$ be the class of all minimal $\lambda_{\alpha c}$-open sets in A and $y \in A \setminus (A_1 \cup A_2 \cup \dots \cup A_m)$. Then there exists a natural number $k \in \{1,2,3, \dots, m\}$, such that $y \in \lambda_{\alpha c}Cl(A_k)$. where λ is λ-regular.

Proof. It follows from Proposition 5.2, that there exists a natural number $k \in \{1,2,3, \dots, m\}$ such that $A_k \subseteq B$ for any $\lambda_{\alpha c}$-open neighborhood B of y. Therefore $\phi \neq A_k \cap A_k \subseteq A_k \cap B$ implies $y \in \lambda_{\alpha c}Cl(A_k)$. This completes the proof.

Proposition 5.4. Let $\phi \neq A$ be a finite $\lambda_{\alpha c}$-open set in a space X and for each $k \in \{1,2,3, \dots, m\}, A_k$ is a minimal $\lambda_{\alpha c}$-open sets in A. If the class $\{A_1, A_2, \dots, A_m\}$ contains all minimal $\lambda_{\alpha c}$-open sets in A, then for any $\phi \neq B_k \subseteq A_k, A \subseteq \lambda_{\alpha c}Cl(B_1 \cup B_2 \cup B_3 \cup \dots \cup B_m)$, where λ is $semi$-regular.

Proof. If A is a minimal $\lambda_{\alpha c}$ -open set, then this is the result of Theorem 3.11 (2). Otherwise, when A is not a minimal $\lambda_{\alpha c}$-open set. If x is any element of $A \setminus (A_1 \cup A_2 \cup \dots \cup A_m)$, then by Theorem 5.3, $x \in \lambda_{\alpha c}Cl(A_1) \cup \lambda_{\alpha c}Cl(A_2) \cup \dots . \cup \lambda_{\alpha c}Cl(A_m)$. Therefore, by Theorem 3.11 (3), we obtain that $A \subseteq \lambda_{\alpha c}Cl(A_1) \cup$

$\lambda_{\alpha c}Cl(A_2) \cup \ldots \cup \lambda_{\alpha c}Cl(A_m) = \lambda_{\alpha c}Cl(B_1) \cup \lambda_{\alpha c}Cl(B_2) \cup \ldots \cup \lambda_{\alpha c}Cl(B_m) = \lambda_{\alpha c}Cl(B_1 \cup B_2 \cup B_3 \cup \ldots \cup B_m)$.

Proposition 5.5. Let $\phi \neq A$ be a finite $\lambda_{\alpha c}$-open set and A_k is a minimal $\lambda_{\alpha c}$-open set in A, for each $k \in \{1,2,3,\ldots,m\}$. If for any $\phi \neq B_k \subseteq A_k, A \subseteq \lambda_{\alpha c}Cl(B_1 \cup B_2 \cup B_3 \cup \ldots \cup B_m)$ then $\lambda_{\alpha c}Cl(A) = \lambda_{\alpha c}Cl(B_1 \cup B_2 \cup B_3 \cup \ldots \cup B_m)$.

Proof. For any $\phi \neq B_k \subseteq A_k$ with $k \in \{1,2,3,\ldots,m\}$, we have $\lambda_{\alpha c}Cl(B_1 \cup B_2 \cup B_3 \cup \ldots \cup B_m) \subseteq \lambda_{\alpha c}Cl(A)$. Also, we have $\lambda_{\alpha c}Cl(A) \subseteq \lambda_{\alpha c}Cl(B_1) \cup \lambda_{\alpha c}Cl(B_2) \cup \ldots \cup \lambda_{\alpha c}Cl(B_m) = \lambda_{\alpha c}Cl(B_1 \cup B_2 \cup B_3 \cup \ldots \cup B_m)$. Therefore, $\lambda_{\alpha c}Cl(A) = \lambda_{\alpha c}Cl(B_1 \cup B_2 \cup B_3 \cup \ldots \cup B_m)$ for any nonempty subset B_k of $A_k$with $k \in \{1,2,3,\ldots,m\}$.

Proposition 5.6. Let $\phi \neq A$ be a finite $\lambda_{\alpha c}$-open set and for each $k \in \{1,2,3,\ldots,m\}, A_k$is a minimal $\lambda_{\alpha c}$-open set in A. If for any $\phi \neq B_k \subseteq A_k$, $\lambda_{\alpha c}Cl(A) = \lambda_{\alpha c}Cl(B_1 \cup B_2 \cup B_3 \cup \ldots \cup B_m)$, then the class $\{A_1, A_2, \ldots, A_m\}$ contains all minimal $\lambda_{\alpha c}$-open sets in A.

Proof. Suppose that C is a minimal $\lambda_{\alpha c}$-open set in A and $C \neq A_k$ for $k \in \{1,2,3,\ldots,m\}$. Then we have $C \cap \lambda_{\alpha c}Cl(A_k) = \phi$ for each $k \in \{1,2,3,\ldots,m\}$. It follows that any element of C is not contained in $\lambda_{\alpha c}Cl(A_1 \cup A_2 \cup \ldots \cup A_m)$.Thisis a contradiction to the fact that $C \subseteq A \subseteq \lambda_{\alpha c}Cl(A) = \lambda_{\alpha c}Cl(B_1 \cup B_2 \cup B_3 \cup \ldots \cup B_m)$. This completes the proof.

Combining Propositions 5.4, 5.5 and 5.6, we have the following theorem:

Theorem 5.7. Let$A$ be a nonempty finite $\lambda_{\alpha c}$-open set and A_k a minimal $\lambda_{\alpha c}$-open set in A for each $k \in \{1,2,3,\ldots,m\}$. Then the following three conditions are equivalent:

(1) The class $\{A_1, A_2, \ldots, A_m\}$ contains all minimal $\lambda_{\alpha c}$-open sets in A.

(2) For any $\phi \neq B_k \subseteq A_k, A \subseteq \lambda_{\alpha c}Cl(B_1 \cup B_2 \cup B_3 \cup \ldots \cup B_m)$.

(3) For any $\phi \neq B_k \subseteq A_k$, $\lambda_{\alpha c}Cl(A) = \lambda_{\alpha c}Cl(B_1 \cup B_2 \cup B_3 \cup \ldots \cup B_m)$, where$\lambda$ is $semi$-regular.

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