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Investigation of Total Branching Ratio and the Lepton Polarization Asymmetries in $B_s \to \phi\ell^+\ell^-$ Decay with the Fourth-Generation SM

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Abstract

In this study, we investigate influences of fourth generation quarks on the total branching ration and the single lepton polarization asymmetry for $B_s \to \phi\ell^+\ell^-$ decay. Calculating the new Wilson coefficients in effective Hamiltonian show that the total branching ratio and the lepton polarization as well as the combined lepton asymmetries are sensitive to the existence of the fourth generation, therefore it can be an effective way to identify the new generation quarks in high energy physics laboratories.

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1 Introduction

Flavor Changing Neutral Current (FCNC) processes in the Standard Model(SM) do not occur at tree level and are strongly suppressed. Many extensions of the SM naturally have FCNC processes that are induced at the loop level. Rare B meson decay arise by the FCNC $b \to s$ transition provides potentially the most sensitive and precise test for the standard model (SM) in the flavor sector at loop level. In the other hand, rare B meson decays very sensitive to the new physics beyond the SM.

One of the efficient ways in establishing new physics beyond the SM is the measurement of the lepton polarization asymmetry for these decays and investigation of standard model with four generation of fermions. Lepton Polarization asymmetry has been studied in the $B \to K^*\ell^+\ell^-$, $B \to X_s\ell^+\ell^-$, $B \to K\ell^+\ell^-$, $B \to \pi(\rho)\ell^+\ell^-$ and $B_s \to \ell^+\ell^-\gamma$ decays [1-6].

The existence of a fourth generation of fermions has not been excluded, although it is strongly constrained by precision measurements of electroweak observables. The restrictions on the new generation of fermions come from the experimental data on the ρ and S Parameters[7]. Therefore, the mass of fourth generation quark (m'_t) considering this data and other constraints lies between 175 GeV and 600 GeV [8, 9].

The pervious works that support the existence of fourth-generation quarks apply this new physics in different fields, for instance Higgs and neutrino physics, cosmology and dark matter [10]–[15]. The fourth quark (t'), like u, c, t quarks, contributes in the $b \to s$ transition at loop level. Clearly, it would change the branching ratio, CP-asymmetry and polarization asymmetries which have been widely studied in baryonic and semileptonic $b \to s$ transition [16]-[21].

In our previous paper[16], we have studied the influences the fourth generation quarks on the Double lepton polarization asymmetry in $B_s \to \phi\ell^+\ell^-$ decay.In this study, we investigate influences of fourth generation quarks on the total branching ration and the single lepton polarization asymmetry for $B_s \to \phi\ell^+\ell^-$ decay.

This paper is organized as follows. In Section II, we drive the matrix element and the differential decay width for $B_s \to \phi\ell^+\ell^-$ in the SM using effective Hamiltonian. The effect of the fourth generation of quarks on the effective Hamiltonian have been presented in section III. In section IV, we calculate the analytic expressions for the lepton polarization asymmetry. Section V devoted to the numerical analysis of the total branching ratio and the lepton polarization as well as combined lepton polarization asymmetries. Finally, we will have our conclusion in section VI.

2 The differential decay width in the Standard Model

For calculating Differential decay rate, it's necessary to obtain effective Hamiltonian. At quark level, the rare decay $B_s \to \phi\ell^+\ell^-$ demonstrate by $b \to s\ell^+\ell^-$ and effective Hamiltonian relevant for this transition cab be written as

$$\mathcal{H}_{\rm eff}(b \to s\ell^+\ell^-) = -\frac{G_F}{\sqrt{2}} V_{tb}V_{ts}^* \sum_{i=1}^{10} C_i(\mu)\mathcal{O}_i(\mu)\,, \tag{1}$$

where $\mathcal{O}_i(\mu)$ the full set operators and the corresponding Wilson coefficients $C_i(\mu)$ are given in [23]. Matrix element for the $b \to s\ell^+\ell^-$ transition by using above effective Hamiltonian as

$$\begin{aligned}
\mathcal{M}(b \to s\ell^+\ell^-) &= < s\ell^+\ell^-|\mathcal{H}_{\rm eff}|b > \\
&= -\frac{G_F}{\sqrt{2}} V_{tb}V_{ts}^* \sum_i C_i^{\rm eff}(\mu) < s\ell^+\ell^-|\mathcal{O}_i|b >^{tree} . \\
&= -\frac{G_F\alpha}{2\pi\sqrt{2}} V_{tb}V_{ts}^* \Bigg[\tilde{C}_9^{\rm eff}\bar{s}\gamma_\mu(1-\gamma_5)b\,\bar{\ell}\gamma_\mu\ell \\
&\qquad + \tilde{C}_{10}^{\rm eff}\bar{s}\gamma_\mu(1-\gamma_5)b\,\bar{\ell}\gamma_\mu\gamma_5\ell \\
&\qquad - 2C_7^{\rm eff}\frac{m_b}{q^2}\bar{s}\sigma_{\mu\nu}q^\nu(1+\gamma_5)b\,\bar{\ell}\gamma_\mu\ell \Bigg],
\end{aligned} \tag{2}$$

where effective Wilson coefficients $C_7^{\rm eff}$, $\tilde{C}_9^{\rm eff}$ and $\tilde{C}_{10}^{\rm eff}$ at μ scale, can be written in the following form [23, 24]:

$$\begin{aligned}
C_7^{\rm eff} &= C_7 - \frac{1}{3}C_5 - C_6 \\
C_{10}^{\rm eff} &= \frac{\alpha}{2\pi}\tilde{C}_{10}^{\rm eff} = C_{10} \\
C_9^{\rm eff} &= \frac{\alpha}{2\pi}\tilde{C}_9^{\rm eff} = C_9 + \frac{\alpha}{2\pi}Y(s).
\end{aligned} \tag{3}$$

where $s = q^2/m_b^2$ and the function $Y(s)$ dedicates the perturbative part due to the one loop matrix elements of four quark operators as [23],

$$\begin{aligned}
Y(s) &= Y_{per}(s) + \frac{3\pi}{\alpha^2}(3C_1 + C_2 + 3C_3 + C_4 + 3C_5 + C_6) \\
&\times \sum_{V_i=\psi_i} \kappa_i \frac{m_{V_i}\Gamma(V_i \to \ell^+\ell^-)}{m_{V_i}^2 - sm_b^2 - im_{V_i}\Gamma_{V_i}},
\end{aligned} \tag{4}$$

where the second term is Breit-Wigner form of the resonance propagator and $Y_{per}(s)$ is long distance contributions which have provenance to the real $c\bar{c}$ intermediate states, i.e., $J/\psi, \psi', \cdots$ that can be written as [25]-[27]

$$\begin{aligned} Y_{per}(s) &= h(\frac{m_c}{m_b}, s)(3C_1 + C_2 + 3C_3 + C_4 + 3C_5 + C_6) \\ &- \frac{1}{2}h(1, s)(4C_3 + 4C_4 + 3C_5 + C_6) \\ &- \frac{1}{2}h(0, s)(C_3 + 3C_4) + \frac{2}{9}(3C_3 + C_4 + 3C_5 + C_6). \end{aligned} \tag{5}$$

the values of C_i and The explicit expressions for h functions can be found in [23] as well as the phenomenological parameters κ_i in Eq.(4) can be determined from experimental measurements of semileptonic B decays

$$\mathcal{B}(B \to K^* V_i \to K^* \ell^+ \ell^-) = \mathcal{B}(B \to K^* V_i)\, \mathcal{B}(V_i \to \ell^+ \ell^-), \tag{6}$$

where the data for the right hand side is given in [28]. For the lowest resonances, J/ψ and ψ' one can use $\kappa = 1.65$ and $\kappa = 2.36$, respectively (see [29]).

Now, By using effective Hamiltonian and relevant Wilson coefficients, we can calculate the transition matrix elements which is vital for computing the decay width and other physical observables of $B_s \to \phi \ell^+ \ell^-$ decay. For this purpose, we need to sandwich effective Hamiltonian between initial hadron state $B(p_{B_s})$ and final hadron state $\phi(p_\phi)$ that can be parameterized in terms of form factors as

$$\begin{aligned} < \phi(p_\phi, \epsilon) \mid \bar{s}\gamma_\mu(1-\gamma_5)b \mid B(p_{B_s}) >= -\frac{2V(q^2)}{m_{B_s} + m_\phi}\epsilon_{\mu\nu\rho\sigma} p_\phi^\rho q^\sigma \epsilon^{*\nu} \\ -i \left[\epsilon_\mu^*(m_{B_s} + m_\phi)A_1(q^2) - (\epsilon^* q)(p_{B_s} + p_\phi)_\mu \frac{A_2(q^2)}{m_{B_s} + m_\phi} \right. \\ \left. - q_\mu(\epsilon^* q)\frac{2m_\phi}{q^2}(A_3(q^2) - A_0(q^2)) \right], \end{aligned} \tag{7}$$

$$\begin{aligned} &\langle \phi(p_\phi, \varepsilon) \left| \bar{s} i \sigma_{\mu\nu} q^\nu (1+\gamma_5) b \right| B(p_{B_s}) \rangle = \\ &\quad 4\epsilon_{\mu\nu\rho\sigma}\varepsilon^{*\nu} p^\rho q^\sigma T_1(q^2) + 2i\left[\varepsilon_\mu^*(m_{B_s}^2 - m_\phi^2) - (p_{B_s} + p_\phi)_\mu(\varepsilon^* q)\right] T_2(q^2) \\ &\quad +2i(\varepsilon^* q)\left[q_\mu - (p_{B_s} + p_\phi)_\mu \frac{q^2}{m_{B_s}^2 - m_\phi^2}\right] T_3(q^2) . \end{aligned} \tag{8}$$

where ε_μ is the polarization vector of meson ϕ and $q = p_{B_s} - p_\phi$ is the momentum transfer as well as $A_3(q^2)$ can be written as a linear combination of the form factors

A_1 and A_2 :

$$A_3(q^2) = \frac{1}{2m_\phi}\left[(m_{B_s}+m_\phi)A_1(q^2) - (m_{B_s}-m_\phi)A_2(q^2)\right] . \quad (9)$$

where $A_3(q^2=0) = A_0(q^2=0)$ (this condition ensures that there is no kinematical singularity in the matrix element at $q^2=0$). The other form factors

$$F(q^2) \in \{V(q^2), A_0(q^2), A_1(q^2), A_2(q^2), A_3(q^2), T_1(q^2), T_2(q^2), T_3(q^2)\} ,$$

are fitted to the the following functions [30, 31]:

$$F(q^2) = \frac{F(0)}{1 - a_F\frac{q^2}{m_{B_s}^2} + b_F(\frac{q^2}{m_{B_s}^2})^2}, \quad (10)$$

where the parameters $F(0)$, a_F and b_F are shown in the TableI .

	$A_0^{B_s\to\phi}$	$A_1^{B_s\to\phi}$	$A_2^{B_s\to\phi}$	$V^{B_s\to\phi}$	$T_1^{B_s\to\phi}$	$T_2^{B_s\to\phi}$	$T_3^{B_s\to\phi}$
$F(0)$	0.382	0.296	0.255	0.433	0.174	0.174	0.125
a_F	1.77	0.87	1.55	1.75	1.82	0.70	1.52
b_F	0.856	−0.061	0.513	0.736	0.825	−0.315	0.377

Table 1: The form factors for $B_s \to \phi\,\ell^+\ell^-$ in a three–parameter fit [32].

The matrix element of the $B_s \to \phi\,\ell^+\ell^-$ decay using above equation can be written as:

$$\begin{aligned}
\mathcal{M}(B_s\to\phi\ell^+\ell^-) &= \frac{G\alpha}{4\sqrt{2}\pi}V_{tb}V_{ts}^* \qquad (11)\\
&\times\Bigg\{\bar{\ell}\gamma^\mu(1-\gamma_5)\ell\Big[-2B_0\epsilon_{\mu\nu\lambda\sigma}\varepsilon^{*\nu}p_\phi^\lambda q^\sigma - iB_1\varepsilon_\mu^* \\
&\qquad + iB_2(\varepsilon^* q)(p_{B_s}+p_\phi)_\mu + iB_3(\varepsilon^* q)q_\mu\Big] \\
&+\bar{\ell}\gamma^\mu(1+\gamma_5)\ell\Big[-2C_1\epsilon_{\mu\nu\lambda\sigma}\varepsilon^{*\nu}p_\phi^\lambda q^\sigma - iD_1\varepsilon_\mu^* \\
&\qquad + iD_2(\varepsilon^* q)(p_{B_s}+p_\phi)_\mu + iD_3(\varepsilon^* q)q_\mu\Big]\Bigg\} ,
\end{aligned}$$

where

$$
\begin{aligned}
B_0 &= (\tilde{C}_9^{\text{eff}} - \tilde{C}_{10}^{\text{eff}})\frac{V}{m_{B_s}+m_\phi} + 4(m_{B_s}+m_s)C_7^{\text{eff}}\frac{T_1}{q^2} , \\
B_1 &= (\tilde{C}_9^{\text{eff}} - \tilde{C}_{10}^{\text{eff}})(m_{B_s}+m_\phi)A_1 + 4(m_{B_s}-m_s)C_7^{\text{eff}}(m_{B_s}^2-m_\phi^2)\frac{T_2}{q^2} , \\
B_2 &= \frac{\tilde{C}_9^{\text{eff}} - \tilde{C}_{10}^{\text{eff}}}{m_{B_s}+m_\phi}A_2 + 4(m_{B_s}-m_s)C_7^{\text{eff}}\frac{1}{q^2}\left[T_2 + \frac{q^2}{m_{B_s}^2-m_\phi^2}T_3\right] , \\
B_3 &= 2(\tilde{C}_9^{\text{eff}} - \tilde{C}_{10}^{\text{eff}})m_\phi\frac{A_3-A_0}{q^2} - 4(m_{B_s}-m_s)C_7^{\text{eff}}\frac{T_3}{q^2} , \\
C_1 &= B_0(\tilde{C}_{10}^{\text{eff}} \to -\tilde{C}_{10}^{\text{eff}}) , \\
D_i &= B_i(\tilde{C}_{10}^{\text{eff}} \to -\tilde{C}_{10}^{\text{eff}}) , \quad (i=1,\ 2,\ 3).
\end{aligned}
$$

From the expression of the matrix element given in Eq. (11), we get the following result for the differential decay width

$$
\frac{d\Gamma^\phi}{d\hat{s}}(B_s \to \phi\ell^+\ell^-) = \frac{G^2\alpha^2 m_{B_s}}{2^{14}\pi^5}\,|V_{tb}V_{ts}^*|^2\,\lambda^{1/2}(1,\hat{r},\hat{s})v\Delta(\hat{s}) , \tag{12}
$$

with

$$
\begin{aligned}
\Delta &= \frac{2}{3\hat{r}_\phi\hat{s}}m_{B_s}^2 Re[-12m_{B_s}^2\hat{m_l}^2\lambda\hat{s}\{(B_3-D_2-D_3)B_1^* - (B_3+B_2-D_3)D_1^*\} \\
&\quad +12m_{B_s}^4\hat{m_l}^2\lambda\hat{s}(1-\hat{r}_\phi)(B_2-D_2)(B_3^*-D_3^*) \\
&\quad +48\hat{m_l}^2\hat{r}_\phi\hat{s}(3B_1D_1^* + 2m_{B_s}^4\lambda B_0C_1^*) \\
&\quad -16m_{B_s}^4\hat{r}_\phi\hat{s}\lambda(\hat{m_l}^2-\hat{s})\{|B_0|^2+|C_1|^2\} \\
&\quad -6m_{B_s}^4\hat{m_l}^2\lambda\hat{s}\{2(2+2\hat{r}_\phi-\hat{s})B_2D_2^* - \hat{s}|(B_3-D_3)|^2\} \\
&\quad -4m_{B_s}^2\lambda\{\hat{m_l}^2(2-2\hat{r}_\phi+\hat{s}) + \hat{s}(1-\hat{r}_\phi-\hat{s})\}(B_1B_2^*+D_1D_2^*) \\
&\quad +\hat{s}\{6\hat{r}_\phi\hat{s}(3+v^2) + \lambda(3-v^2)\}\{|B_1|^2+|D_1|^2\} \\
&\quad -2m_{B_s}^4\lambda\{\hat{m_l}^2[\lambda-3(1-\hat{r}_\phi)^2] - \lambda\hat{s}\}\{|B_2|^2+|D_2|^2\}],
\end{aligned}
$$

where $\hat{s} = q^2/m_{B_s}^2$, $\hat{r}_\phi = m_\phi^2/m_{B_s}^2$ and $\lambda(a,b,c) = a^2+b^2+c^2-2ab-2ac-2bc$, $\hat{m}_\ell = m_\ell/m_{B_s}$, $v = \sqrt{1-4\hat{m}_\ell^2/\hat{s}}$ is the final lepton velocity.

3 The Effects of the Fourth-Generation

The new effective hamiltonian with existence of fourth generation quarks as:

$$\mathcal{H}_{\rm eff} = -\frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i^{\rm new}(\mu) \mathcal{O}_i(\mu), \qquad (13)$$

Where $C_i^{\rm new}(\mu)$ are

$$C_i^{\rm new}(\mu) = C_i(\mu) + \frac{\lambda_{t'}}{\lambda_t} C_i^{\rm SM4}(\mu), \qquad i = 1 \ldots 10. \qquad (14)$$

Where $\lambda_f = V_{fb}^* V_{fs} = r_{sb} e^{i\phi_{sb}}$ and $\lambda_{t'}$ can be parameterized as:

$$\lambda_{t'} = V_{t'b} V_{t's}^* = r_{sb} e^{i\phi_{sb}}. \qquad (15)$$

In the above equation, it is clear that the insertion of fourth generation in the $\mathcal{H}_{\rm eff}$ not lead to new operators and all Wilson coefficients receive additional terms as$\frac{\lambda_{t'}}{\lambda_t} C_i^{\rm SM4}(\mu)$ either via virtual exchange of the fourth-generation up type quark $t'(C_3, ..., C_{10})$ or via using the unitarity of CKM matrix (C_1, C_2). The unitary quark mixing matrix is now 4×4 satisfy the relation

$$\lambda_u + \lambda_c + \lambda_t + \lambda_{t'} = 0. \qquad (16)$$

As a result, for the $m_{t'} \to m_t$ or $\lambda_{t'} \to 0$ the factor $\lambda_t C_i^{\rm new}$ should be modified to $\lambda_t C_i$ as required by the GIM mechanism as Eq.(29):

$$\begin{aligned} \lambda_t C_i^{\rm new} = \lambda_t C_i + \lambda_{t'} C_i^{\rm SM4} &= -(\lambda_u + \lambda_c) C_i + \lambda_{t'} (C_i^{\rm SM4} - C_i) \\ &= -(\lambda_u + \lambda_c) C_i \\ &= \lambda_t C_i. \end{aligned} \qquad (17)$$

Now by using the above effective Hamiltonian, we can recalculate the one-loop matrix elements of $b \to s\ell^+\ell^-$ by replacing $C_i^{\rm eff}(\tilde{C}_i^{\rm eff})$ with $C_i^{\rm eff\,new}(\tilde{C}_i^{\rm eff\,new})$ in Eq.(2), where $C_i^{\rm eff\,new}$ and $\tilde{C}_i^{\rm eff\,new}$ are given as:

$$\begin{aligned} C_i^{\rm eff\,new}(\mu) &= C_i^{\rm eff}(\mu) + \frac{\lambda_{t'}}{\lambda_t} C_i^{\rm eff\ SM4}(\mu), \qquad i = 7, \\ \tilde{C}_i^{\rm eff\,new}(\mu) &= \tilde{C}_i^{\rm eff}(\mu) + \frac{\lambda_{t'}}{\lambda_t} \tilde{C}_i^{\rm eff\ SM4}(\mu), \qquad i = 9, 10. \end{aligned} \qquad (18)$$

Here the effective Wilson coefficients $C_i^{\rm eff\,SM4}$ and $\tilde{C}_i^{\rm eff\,SM4}$ are defined in the same way as Eqs.(3) by substituting C_i with $C_i^{\rm SM4}$. It is worth nothing that the explicit forms of $C_i^{\text{eff SM4}}$ and $\tilde{C}_i^{\text{eff SM4}}$ can also be found from the corresponding Wilson coefficients in SM by replacing $m_{t'} \to m_t$ [23]. Consequently, we can reobtain the differential decay width for $B_s \to \phi\ell^+\ell^-$ decay in the presence of the fourth-generation and use it in next section for calculating of the CP-violating asymmetry.

4 Lepton polarization of $B_s \to \phi\ell^+\ell^-$

Having obtained the differential decay width for $B_s \to \phi\ell^+\ell^-$, we can now obtain the single lepton polarization asymmetries. For this purpose, we must first define the orthogonal unit vectors $s^{\pm\mu}$ in the rest frame of leptons, where i=L,N or T to correspond to the longitudinal, normal and transversal polarization directions, respectively:

$$\begin{aligned}
s_L^{-\mu} &= (0, \vec{e}_L^{\,-}) = \left(0, \frac{\vec{p}_-}{|\vec{p}_-|}\right), & s_L^{+\mu} &= (0, \vec{e}_L^{\,+}) = \left(0, \frac{\vec{p}_+}{|\vec{p}_+|}\right), \\
s_N^{-\mu} &= (0, \vec{e}_N^{\,-}) = \left(0, \frac{\vec{p}_\phi \times \vec{p}_-}{|\vec{p}_\phi \times \vec{p}_-|}\right), & s_N^{+\mu} &= (0, \vec{e}_N^{\,+}) = \left(0, \frac{\vec{p}_\phi \times \vec{p}_+}{|\vec{p}_\phi \times \vec{p}_+|}\right), \\
s_T^{-\mu} &= (0, \vec{e}_T^{\,-}) = \left(0, \vec{e}_N^{\,-} \times \vec{e}_L^{\,-}\right), & s_T^{+\mu} &= (0, \vec{e}_T^{\,+}) = \left(0, \vec{e}_N^{\,+} \times \vec{e}_L^{\,+}\right).
\end{aligned} \tag{19}$$

Where $\vec{p}_\mp$ and $\vec{p}_\phi$ are the three–momenta of the leptons $\ell_\pm$ and ϕ meson, respectively. The longitudinal unit vectors is boosted to the CM frame of leptons by Lorenz transformation:

$$\left(s_L^{-\mu}\right)_{CM} = \left(\frac{|\vec{p}_-|}{m_\ell}, \frac{E\vec{p}_-}{m_\ell\,|\vec{p}_-|}\right), \qquad \left(s_L^{+\mu}\right)_{CM} = \left(\frac{|\vec{p}_-|}{m_\ell}, -\frac{E\vec{p}_-}{m_\ell\,|\vec{p}_-|}\right). \tag{20}$$

The polarization asymmetries can now be calculated using the spin projector $\frac{1}{2}(1 + \gamma_5 \not{s}_i^-)$ for ℓ^- and the spin projector $\frac{1}{2}(1 + \gamma_5 \not{s}_i^+)$ for ℓ^+.

Considering the above explanations, we can define the lepton polarization asymmetries as in:

$$P_i^{(\mp)}(q^2) = \frac{\dfrac{d\Gamma}{d\hat{s}}(\vec{n}^{(\mp)} = \vec{e}_i^{\,(\mp)}) - \dfrac{d\Gamma}{d\hat{s}}(\vec{n}^{(\mp)} = -\vec{e}_i^{\,(\mp)})}{\dfrac{d\Gamma}{d\hat{s}}(\vec{n}^{(\mp)} = \vec{e}_i^{\,(\mp)}) + \dfrac{d\Gamma}{d\hat{s}}(\vec{n}^{(\mp)} = -\vec{e}_i^{\,(\mp)})}, \tag{21}$$

where i=L, N, T refer to the longitudinal, normal and transversal polarization that after the calculation as:

$$\begin{aligned}
P_L^- &= \frac{4}{\Delta} m_B^2 v \Bigg\{ \frac{1}{3r} \lambda^2 m_B^4 \Big[|B_2|^2 - |D_2|^2 \Big] \\
&+ \frac{8}{3} \lambda m_B^4 s \Big[|A_1|^2 - |C_1|^2 \Big] \\
&- \frac{2}{3r} \lambda m_B^2 (1 - r - s) \Big[Re(B_1 B_2^*) - Re(D_1 D_2^*) \Big] \\
&+ \frac{1}{3r} (\lambda + 12rs) \Big[|B_1|^2 - |D_1|^2 \Big] \Bigg\} ,
\end{aligned} \tag{22}$$

$$\begin{aligned}
P_L^+ &= \frac{4}{\Delta} m_B^2 v \Bigg\{ - \frac{1}{3r} \lambda^2 m_B^4 \Big[|B_2|^2 - |D_2|^2 \Big] \\
&- \frac{8}{3} \lambda m_B^4 s \Big[|A_1|^2 - |C_1|^2 \Big] \\
&+ \frac{2}{3r} \lambda m_B^2 (1 - r - s) \Big[Re(B_1 B_2^*) - Re(D_1 D_2^*) \Big] \\
&- \frac{1}{3r} (\lambda + 12rs) \Big[|B_1|^2 - |D_1|^2 \Big] \Bigg\} ,
\end{aligned} \tag{23}$$

$$\begin{aligned}
P_T^- &= \frac{\pi}{\Delta} m_B \sqrt{s\lambda} \Bigg\{ - 8 m_B^2 m_\ell Re[(A_1 + C_1)(B_1^* + D_1^*)] \\
&+ \frac{1}{r} m_B^2 m_\ell (1 + 3r - s) \Big[Re(B_1 D_2^*) - Re(B_2 D_1^*) \Big] \\
&+ \frac{1}{rs} m_\ell (1 - r - s) \Big[|B_1|^2 - |D_1|^2 \Big] \\
&- \frac{1}{r} m_B^2 m_\ell (1 - r - s)\, Re[(B_1 + D_1)(B_3^* - D_3^*)] \\
&+ \frac{1}{rs} m_B^4 m_\ell (1 - r) \lambda \Big[|B_2|^2 - |D_2|^2 \Big] \\
&+ \frac{1}{r} m_B^4 m_\ell \lambda\, Re[(B_2 + D_2)(B_3^* - D_3^*)] \\
&- \frac{1}{rs} m_B^2 m_\ell [\lambda + (1 - r - s)(1 - r)] \Big[Re(B_1 B_2^*) - Re(D_1 D_2^*) \Big] \Bigg\} ,
\end{aligned} \tag{24}$$

$$
\begin{aligned}
P_T^+ &= \frac{\pi}{\Delta} m_B \sqrt{s\lambda} \Bigg\{ -8 m_B^2 m_\ell Re[(A_1 + C_1)(B_1^* + D_1^*)] \\
&- \frac{1}{r} m_B^2 m_\ell (1 + 3r - s) \Big[Re(B_1 D_2^*) - Re(B_2 D_1^*) \Big] \\
&- \frac{1}{rs} m_\ell (1 - r - s) \Big[|B_1|^2 - |D_1|^2 \Big] \\
&+ \frac{1}{r} m_B^2 m_\ell (1 - r - s)\, Re[(B_1 + D_1)(B_3^* - D_3^*)] \\
&- \frac{1}{rs} m_B^4 m_\ell (1 - r)\lambda \Big[|B_2|^2 - |D_2|^2 \Big] \\
&- \frac{1}{r} m_B^4 m_\ell \lambda\, Re[(B_2 + D_2)(B_3^* - D_3^*)] \\
&+ \frac{1}{rs} m_B^2 m_\ell [\lambda + (1 - r - s)(1 - r)] \Big[Re(B_1 B_2^*) - Re(D_1 D_2^*) \Big] \Bigg\} ,
\end{aligned} \tag{25}
$$

$$
\begin{aligned}
P_N^- &= \frac{1}{\Delta} \pi v m_B^3 \sqrt{s\lambda} \Bigg\{ 8 m_\ell Im[(B_1^* C_1) + (A_1^* D_1)] \\
&+ \frac{1}{r} m_B^2 m_\ell \lambda\, Im[(B_2 - D_2)(B_3^* - D_3^*)] \\
&- \frac{1}{r} m_\ell\, (1 + 3r - s)\, Im[(B_1 - D_1)(B_2^* - D_2^*)] \\
&- \frac{1}{r} m_\ell\, (1 - r - s)\, Im[(B_1 - D_1)(B_3^* - D_3^*)] \Bigg\} ,
\end{aligned} \tag{26}
$$

$$
\begin{aligned}
P_N^+ &= \frac{1}{\Delta} \pi v m_B^3 \sqrt{s\lambda} \Bigg\{ -8 m_\ell Im[(B_1^* C_1) + (A_1^* D_1)] \\
&+ \frac{1}{r} m_B^2 m_\ell \lambda\, Im[(B_2 - D_2)(B_3^* - D_3^*)] \\
&- \frac{1}{r} m_\ell\, (1 + 3r - s)\, Im[(B_1 - D_1)(B_2^* - D_2^*)] \\
&- \frac{1}{r} m_\ell\, (1 - r - s)\, Im[(B_1 - D_1)(B_3^* - D_3^*)] \Bigg\} .
\end{aligned} \tag{27}
$$

5 Numerical analysis

In this section, we will analyze the dependence of the single lepton polarizations to the mass of fourth quark (mt') and the product of quark mixing matrix elements $(V_{t'b}V^*_{t's} = r_{sb}e^{i\phi_{sb}})$. The main input parameters in the calculations are the form factors, which are the predictions of light cone QCD sum rule method [30, 31],as pointed out in section II. The other input parameters we use in our numerical calculations as follow:

$$m_{B_s} = 5.37\,\text{GeV}\,,\; m_b = 4.8\,\text{GeV}\,,\; m_c = 1.5\,\text{GeV}\,,\; m_\tau = 1.77\,\text{GeV}\,,$$
$$m_\mu = 0.105\,\text{GeV},\; m_\phi = 1.020\,\text{GeV}\;,\; |V_{tb}V^*_{ts}| = 0.0385\,,\; \alpha^{-1} = 129\,,$$
$$G_f = 1.166\times 10^{-5}\,\text{GeV}^{-2}\,,\; \tau_{B_s} = 1.46\times 10^{-12}\,s\,. \tag{28}$$

In order to present a quantitative analysis of the lepton polarization asymmetries, the values of fourth-generation parameters $(m_{t'}, r_{sb}, \phi_{sb})$ are needed. We include the bound on $r_{sb} \sim \{0.01 - 0.03\}$ for $\phi_{sb} \sim \{0^\circ - 360^\circ\}$ and $m_{t'} \sim \{200 - 600\}$ GeV using the experimental values of $B \to X_s\gamma$ and $B \to X_s\ell^+\ell^-$ decays[21, 32]. In the other hand, considering the B_s mixing, which is in terms of the Δm_{B_s} a sharp restriction on ϕ_{sb} has been obtained $(\phi_{sb} \sim 90^\circ)$[33]. Accordingly, this new parameters taking into account all the above constraints can be determine as:
$r_{sb} = \{0.01, 0.02, 0.03\}$, $\phi_{sb} = \{60^\circ, 90^\circ, 120^\circ\}$, $m_{t'} = 175 \le m_{t'} \le 600$.
Now before performing numerical analysis, we should solve a problem about dependencies of the lepton polarizations formulas (P_i) on both $\hat{s}$ and new parameters $(m_{t'}, r_{sb}, \phi_{sb})$, because it may be experimentally difficult to investigate these dependencies at the same time. One way to deal with this problem is to integrate over q^2 and study the averaged lepton polarization asymmetries. The total branching ratio and average of P_i over q^2 are defined as:

$$B_r = \int_{4\hat{m}_\ell^2}^{(1-\sqrt{\hat{r_\phi}})^2} \frac{d\mathcal{B}}{d\hat{s}}d\hat{s}\;. \tag{29}$$

$$\langle P_i\rangle = \frac{\displaystyle\int_{4\hat{m}_\ell^2}^{(1-\sqrt{\hat{r_\phi}})^2} P_i\frac{d\mathcal{B}}{d\hat{s}}d\hat{s}}{B_r}\;. \tag{30}$$

Figs.1-7 depict the dependence of the total branching ratio and the averaged lepton polarizations $\langle P_L^-\rangle$, $\langle P_N^-\rangle$,$\langle P_N^-\rangle$, $\langle P_N^- - P_N^+\rangle$, $\langle P_T^- - P_T^+\rangle$ and $\langle P_T^- + P_T^+\rangle$ for various r_{sb} in terms of $m_{t'}$. From these figures, we derive the following results.

- FIG.1: For both μ and τ channels, the values of B_r strongly dependents on the fourth quark mass $(m_t{}')$ and the product of quark mixing matrix elements (r_{sb}). Furthermore, B_r is increasing function of r_{sb} and ϕ_{sb} for both channels while it is increasing and decreasing function of $(m_t{}')$ if the $\phi_{sb} = 60°$.
- FIG.2: Although, $\langle P_L^- \rangle$ strongly sensitive to $m_{t'}$ and r_{sb} for both μ and τ channels. But, its magnitude is a decreasing function of $m_{t'}$. So, the existence of fourth generation of quarks will suppress the magnitude of $\langle P_L^- \rangle$.
- FIG.3: In the $\phi_{sb} = 60°, 120°$, the magnitude of $\langle P_T^- \rangle$ is very sensitive to the fourth quark mass $(m_{t'})$ for both μ and τ channels, while it is less sensitive to $m_{t'}$ and r_{sb} parameters if the $\phi_{sb} = 90°$. In the other hand, the situation for the sign of $\langle P_T^- \rangle$ is very interesting, since for fixed values of $m_{t'}$ and r_{sb}, and increase in ϕ_{sb} changes the sign of $\langle P_T^- \rangle$. So, the study of the magnitude and the sign of this asymmetry can serve as good tests for discovering the new physics beyond the SM. It should also be mentioned that, $\langle P_T^- \rangle$ is increasing and decreasing function of $m_{t'}$ for $\phi_{sb} = 60°$ for both case μ and τ.
- FIG.4: Taking into account the 4th generation, the value of $\langle P_N^- \rangle$ exceeds the SM result 8 and 6 times for τ and μ cases, respectively. Furthermore, in μ and τ channels by increasing r_{sb} and keeping the values of ϕ_{sb} fixed, the maximum deviation from SM occurs at smaller values of $m_{t'}$. This result can be interesting since the maximum deviation from SM happens for $r_{sb} \sim \{0.02 - 0.03\}$ and $m_{t'} \sim \{300 - 400\}$GeV. Therefore, the new generation has a chance to be observed around $m_t' \sim \{300 - 400\}$ GeV.
- FIG.5: Numerical calculations show that the combined $\langle P_N^- - P_N^+ \rangle$ have intensive dependency on the fourth-generation parameter $m_{t'}$ for both μ and τ channels. But this dependency in τ case is greater than μ case. Similar to the $\langle P_N^- \rangle$ the maximum deviation from SM happens for $r_{sb} \sim \{0.02 - 0.03\}$ and $m_{t'} \sim \{300 - 400\}$GeV. Therefor The measurement $\langle P_N^- - P_N^+ \rangle$ and $\langle P_N^- \rangle$,especially, for τ case can used as a good tool when looking for the fourth generation of quarks.
- FIG.6: $\langle P_T^- - P_T^+ \rangle$ are quite sensitive to the existence of the SM4 parameters $(m_{t'}, r_{sb})$ for both cases. Moreover, the maximum deviation from SM in τ channel is much more than that in μ channel. Furthermore, its magnitude is decreasing for increasing values of $m_{t'}$ and r_{sb}.
- FIG.7: Regarding the 4th generation, the values of $\langle P_N^- + P_N^+ \rangle$ show strongly

dependence on the fourth-generation parameters $(m_{t'}, r_{sb}, \phi_{sb})$ for both μ and τ channels. But, it is increasing and decreasing for the $\phi_{sb} = 60^\circ$ in both channels.

Our numerical analysis show that not only the lepton polarization but also the combinations of them can be used as a good tool to look for new Physics effects.

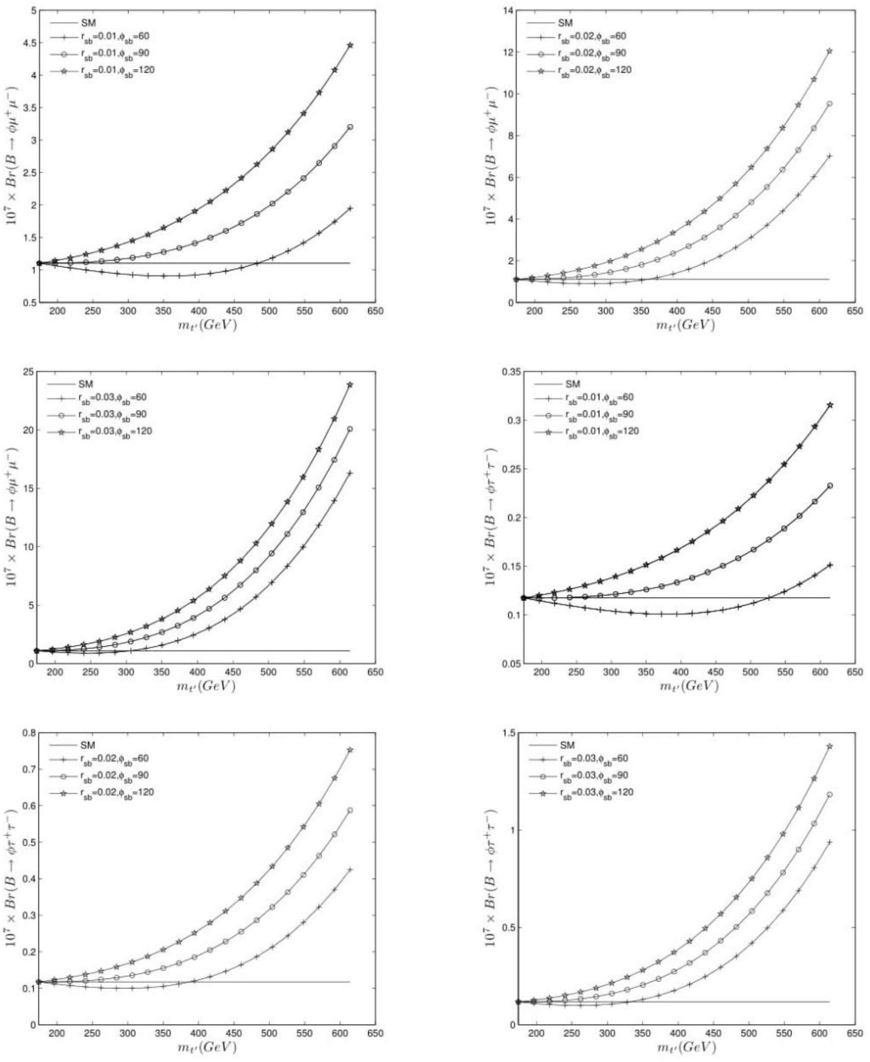

Figure 1: The dependence of the B_r on the fourth-generation quark mass $m_{t'}$ at three different values of $\phi_{sb} = \{60^\circ,\ 90^\circ,\ 120^\circ\}$ and $r_{sb} = \{0.01,\ 0.02,\ 0.03\}$ for the μ and τ channels.

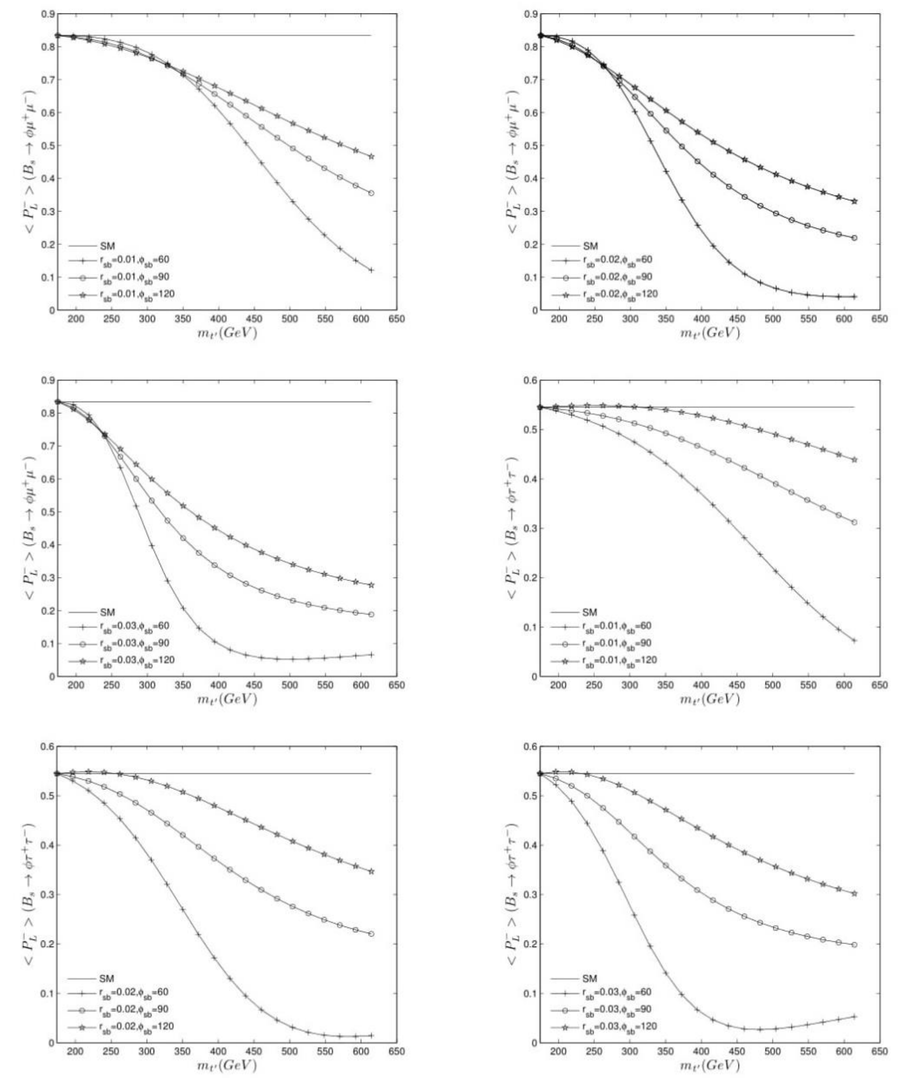

Figure 2: The dependence of the $\langle P_L^- \rangle$ on the fourth-generation quark mass $m_{t'}$ at three different values of $\phi_{sb} = \{60^\circ,\ 90^\circ,\ 120^\circ\}$ and $r_{sb} = \{0.01,\ 0.02,\ 0.03\}$ for the μ and τ channels.

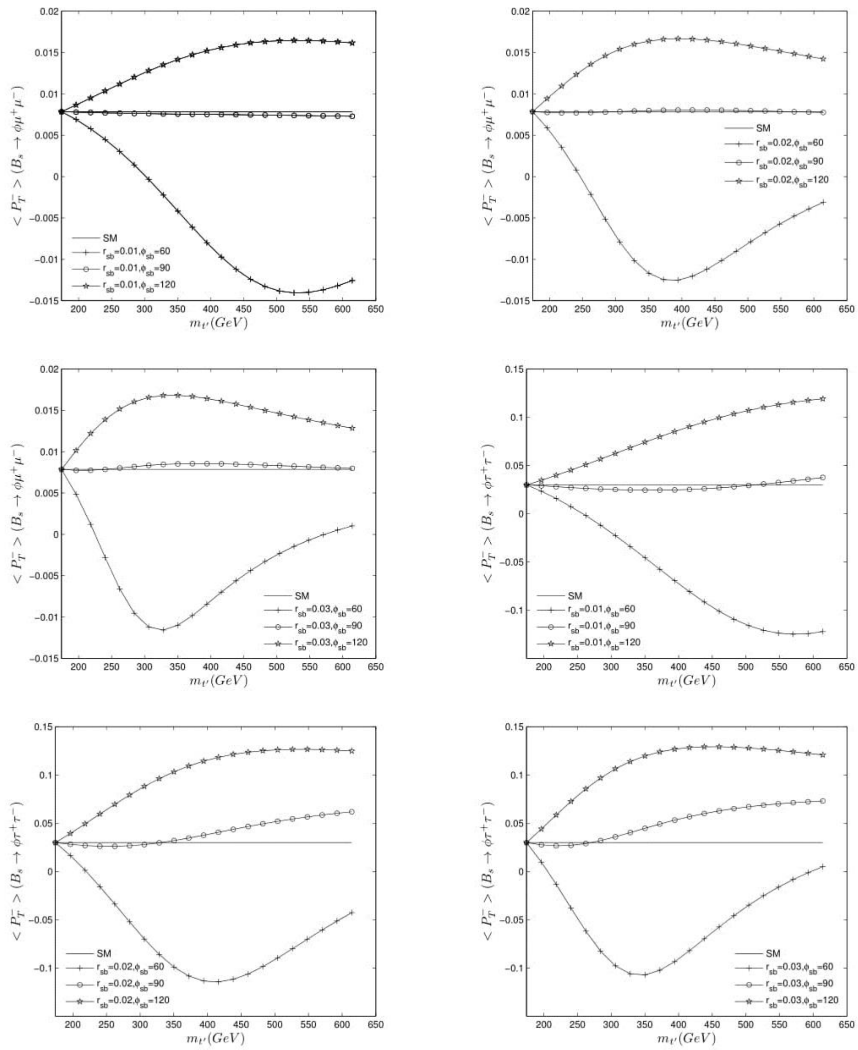

Figure 3: The dependence of the $\langle P_T^- \rangle$ on the fourth-generation quark mass $m_{t'}$ at three different values of $\phi_{sb} = \{60^\circ,\ 90^\circ,\ 120^\circ\}$ and $r_{sb} = \{0.01,\ 0.02,\ 0.03\}$ for the μ and τ channels.

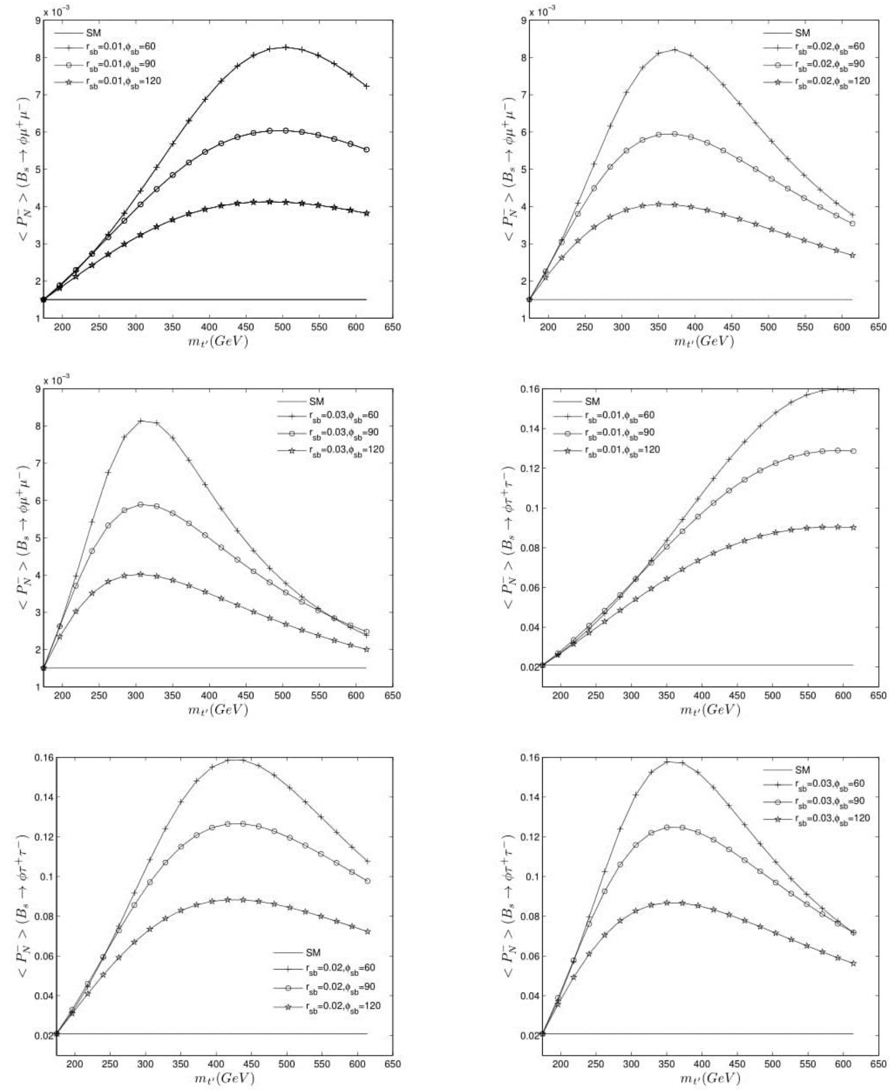

Figure 4: The dependence of the $\left\langle P_N^- \right\rangle$ on the fourth-generation quark mass $m_{t'}$ at three different values of $\phi_{sb} = \{60^\circ,\ 90^\circ,\ 120^\circ\}$ and $r_{sb} = \{0.01,\ 0.02,\ 0.03\}$ for the μ and τ channels.

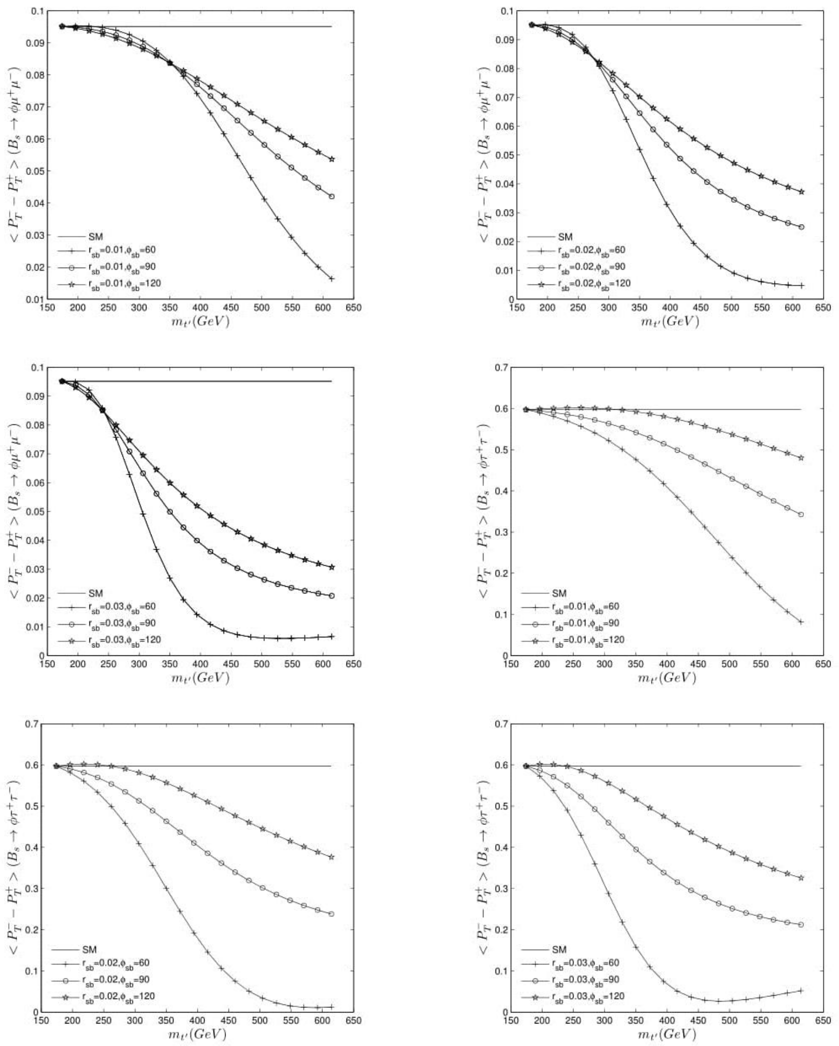

Figure 6: The dependence of the $\left\langle P_T^- - P_T^+ \right\rangle$ on the fourth-generation quark mass $m_{t'}$ at three different values of $\phi_{sb} = \{60^\circ,\ 90^\circ,\ 120^\circ\}$ and $r_{sb} = \{0.01,\ 0.02,\ 0.03\}$ for the μ and τ channels.

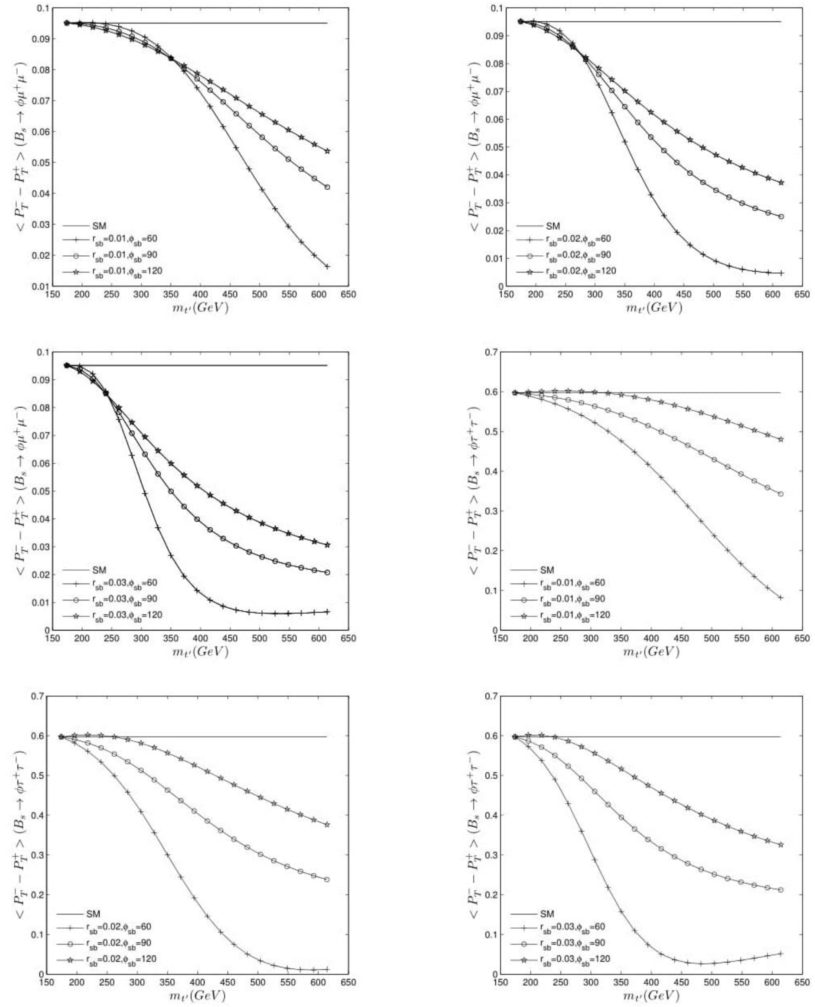

Figure 6: The dependence of the $\langle P_T^- - P_T^+ \rangle$ on the fourth-generation quark mass $m_{t'}$ at three different values of $\phi_{sb} = \{60^\circ,\ 90^\circ,\ 120^\circ\}$ and $r_{sb} = \{0.01,\ 0.02,\ 0.03\}$ for the μ and τ channels.

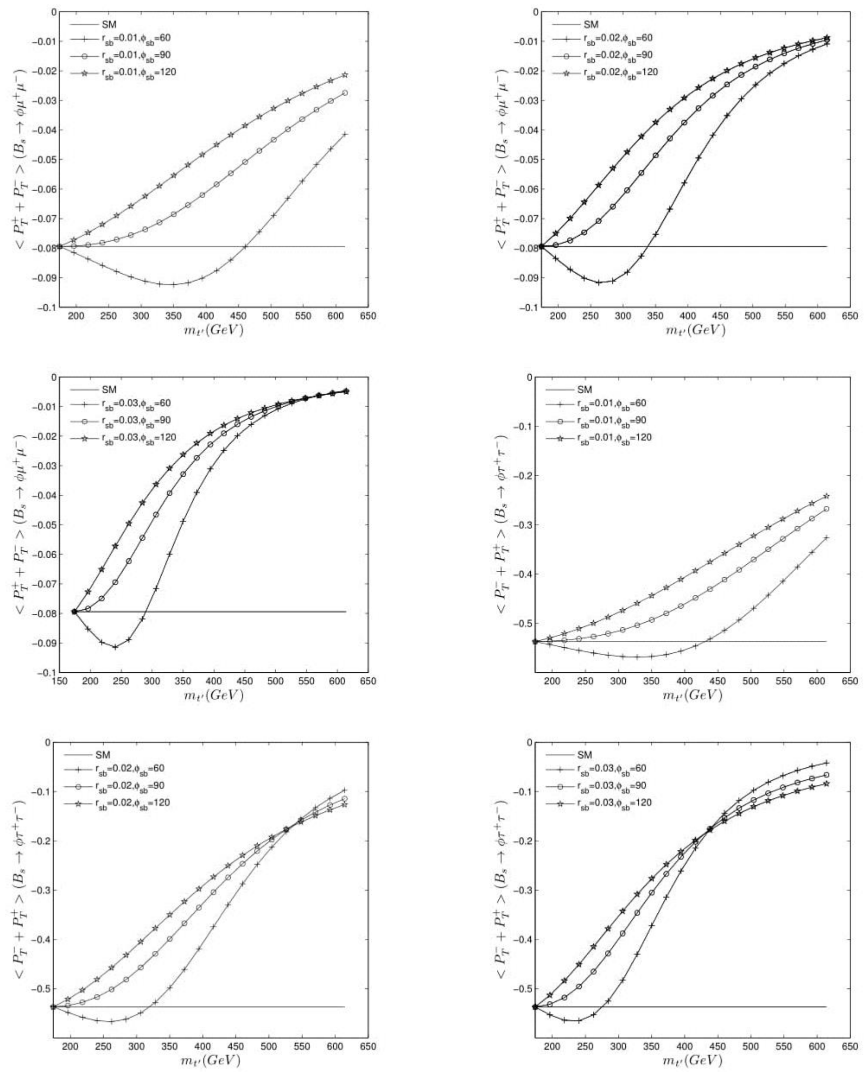

Figure 7: The dependence of the $\left\langle P_N^- + P_N^+ \right\rangle$ on the fourth-generation quark mass $m_{t'}$ at three different values of $\phi_{sb} = \{60^\circ,\ 90^\circ,\ 120^\circ\}$ and $r_{sb} = \{0.01,\ 0.02,\ 0.03\}$ for the μ and τ channels.

6 Conclusion

To conclude, we present a the analysis with detail of the total branching ratio and the lepton polarization asymmetry for $B_s \to \phi\ell^+\ell^-$ in SM with fourth generation of quarks. Both physical observable demonstrated strongly dependence to fourth generation parameters which can be detected at the LHC. The lepton-anti-lepton combined asymmetries as well as depicted high sensitivity to parameters of fourth generation SM. In particular, the results could be used to indirect search to look for fourth generation of quarks and physics beyond the SM.

7 Acknowledgment

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