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$\lambda_{\beta c}$-Connected Spaces and $\lambda_{\beta c}$-Components

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Abstract

In this paper, we define and study a new type of connected spaces called $\lambda_{\beta c}$-connected space. It is remarkable that the class of λ-connected spaces is a subclass of the class of $\lambda_{\beta c}$-connected spaces. We discuss some characterizations and properties of $\lambda_{\beta c}$-connected spaces , $\lambda_{\beta c}$-components and $\lambda_{\beta c}$-locally connected spaces.

1. Introduction

The study of semi-open sets and their properties was initiated by N. Levine [1] in 1963. In [2], S.F.Namiq defined an operation λ on the family of semi open sets in a topological space called s-operation via this operation, he defined λ-open sets. By using λ-open and semi closed set also S.F.Namiq in [3], defined $\lambda_{\beta c}$-open set and also investigated several properties of $\lambda_{\beta c}$-derived, $\lambda_{\beta c}$-interior and $\lambda_{\beta c}$-closure points in topological spaces, moreover In [4], S.F.Namiq defined λ-connected spaces by using λ-open sets. In [5], Furthermore S.F. Namiq defined λ_c-connected spaces via λ_c-open sets. S. Willard in [6], obtained some analogous properties of connectedness for $\lambda_{\beta c}$-connectedness . Throughout the present paper, a topological space is denoted by (X,τ) or simply by X .

2. Preliminaries

First, we recall some definitions and results used in this paper. For any subset A of X, the closure and the interior of A are denoted by $Cl(A)$ and $Int(A)$, respectively. A subset A of a space X is said to be semi open [1] if $A \subseteq Cl(Int(A))$. The complement of a semi open set is said to be semi closed [1]. The family of all semi open (resp. semi closed) sets in a space X is denoted by $SO(X,\tau)$ or $SO(X)$ (resp. $SC(X,\tau)$ or $SC(X)$). A space X is said to be s-connected [7],if it is not the union of two nonempty disjoint semi open subsets of X. A subset A of a topological space X is said to be β-open [8],if $A \subseteq Cl(Int(Cl(A)))$. The complement of a β-open set is said to be β-closed . The family of all β-open (resp. β-closed) sets in a topological space (X,τ) is denoted by $\beta O(X,\tau)$ or $\beta O(X)$ (resp. $\beta C(X,\tau)$ or $\beta C(X)$). We consider $\lambda : SO(X) \rightarrow P(X)$ as a function defined on $SO(X)$ into the power set of X, $P(X)$ and λ is called an s-operation if $V \subseteq \lambda(V)$, for each semi open set V. It is assumed that $\lambda(\phi) = \phi$ and $\lambda(X) = X$, for any s-operation λ. Let X be a space and $\lambda : SO(X) \rightarrow P(X)$ be an s-operation, then a subset A of X is called a λ-open set [2], which is equivalent to λ_s-open set [9],if for each $x \in A$, there exists a semi open set U such that $x \in U$ and $\lambda(U) \subseteq A$. The complement of a λ-open set is said to be λ-closed. The family of all λ-open (resp., λ-closed) subsets of a space X is denoted by $SO_{\lambda}(X,\tau)$ or $SO_{\lambda}(X)$ (resp, $SC_{\lambda}(X,\tau)$ or $SC_{\lambda}(X)$), then a λ-open subset A of X is called a λ_c-open set [2],if for each $x \in A$, there exists a closed set F such that $x \in F \subseteq A$. The family of all λ_c-open (resp., λ_c-closed) subsets of a space X is denoted by $SO_{\lambda c}(X,\tau)$ or $SO_{\lambda c}(X)$ (resp, $SC_{\lambda c}(X,\tau)$ or $SC_{\lambda c}(X)$).

Now, we recall some definitions and restate some known results which will be used in the sequel.

Definition 2.1[2]. Let X be a space and $\lambda : SO(X) \to P(X)$ be an s-operation, then a subset A of X is called a λ-open set if for each $x \in A$ there exists a semi open set U such that $x \in U$ and $\lambda(U) \subseteq A$.

The complement of a λ-open set is called λ-closed. The family of all λ-open (resp., λ-closed) subsets of a topological space (X,τ) is denoted by $SO_{\lambda}(X,\tau)$ or $SO_{\lambda}(X)$ (resp. , $SC_{\lambda}(X,\tau)$ or $SC_{\lambda}(X)$).

Definition 2.2[2]. A λ-open subset A of X is called a λ_c-open set if for each $x \in A$ there exists a closed set F such that $x \in F \subseteq A$. The family of all λ_c-open (resp., λ_c-closed) subsets of a space X is denoted by $SO_{\lambda c}(X,\tau)$ or $SO_{\lambda c}(X)$(resp, $SC_{\lambda c}(X,\tau)$ or $SC_{\lambda c}(X)$).

Definition 2.3 [3]. A λ-open subset A of X is called a $\lambda_{\beta c}$-open set if for each $x \in A$, there exists a β-closed set F such that $x \in F \subseteq A$. The family of all $\lambda_{\beta c}$-open (resp., $\lambda_{\beta c}$-closed) subsets of a space X is denoted by $SO_{\lambda_{\beta c}}(X,\tau)$ or $SO_{\lambda_{\beta c}}(X)$(resp, $SC_{\lambda_{\beta c}}(X,\tau)$ or $SC_{\lambda_{\beta c}}(X)$).

Proposition 2.4 [3]. For a space X, $SO_{\lambda c}(X) \subseteq SO_{\lambda_{\beta c}}(X) \subseteq SO_{\lambda}(X) \subseteq SO(X)$.

The following examples show that the converse of the above proposition may not be true in general.

Example 2.5. Let $X = \{a,b,c\}$, and $\tau = \{\phi,\{a\},X\}$. Define an s-operation $\lambda : SO(X) \to P(X)$ as follows:

$$\lambda(A) = \begin{cases} A & \text{if } b \in A \\ X & \text{otherwise} \end{cases}.$$

Here, we have $\{a,c\}$ is a semi open set, but it is not λ-open. And also we have $\{a,b\}$ is a λ-open set but it is a $\lambda_{\beta c}$-open set, but not λ_c-open.

Example 2.6. Let $X=\{a,b,c\}$, and $\tau=\{\phi,\{a\},\{b\},\{a,b\},X\}$. We define an s-operation $\lambda:SO(X)\to P(X)$ as:

$$\lambda(A)=\begin{cases} A & \text{if } A=\{b\} \\ X & \text{otherwise} \end{cases}.$$

Here, we have $\{b\}$ is a $\lambda_{\beta c}$-open set, but it is not λ_c-open.

Definition 2.17 [9]. Let X be a space, an s-operation λ is said to be s-regular if for every semi open sets U and V containing $x\in X$, there exists a semi open set W containing x such that $\lambda(W)\subseteq\lambda(U)\cap\lambda(V)$.

Definition 2.8 .[3]. Let X be a space and A a subset of X .Then:

(1) The $\lambda_{\beta c}$-closure of A ($\lambda_{\beta c}Cl(A)$) is the intersection of all $\lambda_{\beta c}$-closed sets which containing A.

(2) The $\lambda_{\beta c}$-interior of A ($\lambda_{\beta c}Int(A)$) is the union of all $\lambda_{\beta c}$-open sets of X which contained in A.

(3) A point $x\in X$ is said to be a $\lambda_{\beta c}$-limit point of A if every $\lambda_{\beta c}$-open set containing x contains a point of A different from x ,and the set of all $\lambda_{\beta c}$-limit points of A is called the $\lambda_{\beta c}$-derived set of A, denoted by $\lambda_{\beta c}D(A)$.

Proposition 2.9.[3]. For each point $x\in X$, $x\in\lambda_{\beta c}Cl(A)$ if and only if $V\cap A\neq\phi$, for every $V\in SO_{\lambda_{\beta c}}(X)$ such that $x\in V$.

Proposition 2.10.[3]. Let $\{A_\alpha\}_{\alpha\in I}$ be any collection of $\lambda_{\beta c}$-open sets in a topological space (X,τ), then $\bigcup_{\alpha\in I} A_\alpha$ is a $\lambda_{\beta c}$-open set.

Example 2.11. Let $X=\{a,b,c\}$ and $\tau=P(X)$ .We define an s-operation $\lambda:SO(X)\to P(X)$ as:

$$\lambda(A)=\begin{cases} A & \text{if } A\neq\{a\},\{b\} \\ X & \text{otherwise}\end{cases}.$$

Now, we have $\{a,b\}$ and $\{b,c\}$ are $\lambda_{\beta c}$-open sets, but $\{a,b\}\cap\{b,c\}=\{b\}$ is not $\lambda_{\beta c}$-open.

Proposition 2.12.[3]. Let λ be an s-operation and s-regular. If A and B are $\lambda_{\beta c}$-open sets in X ,then $A\cap B$ is also a $\lambda_{\beta c}$-open set.

Proposition 2.13.[3]**.** Let X be a space and $A\subseteq X$ .Then A is a $\lambda_{\beta c}$-closed subset of X if and only if $\lambda_{\beta c}D(A)\subseteq A$.

Proposition 2.14.[3]. For subsets A,B of a space X , the following statements are true.

(1) $A\subseteq\lambda_{\beta c}Cl(A)$.

(2) $\lambda_{\beta c}Cl(A)$ is a $\lambda_{\beta c}$-closed set in X .

(3) $\lambda_{\beta c}Cl(A)$ is a smallest $\lambda_{\beta c}$-closed set, which contain A.

(4) A is a $\lambda_{\beta c}$-closed set if and only if $A=\lambda_{\beta c}Cl(A)$.

(5) $\lambda_{\beta c}Cl(\phi)=\phi$ and $\lambda_{\beta c}Cl(X)=X$.

(6) If A and B are subsets of the space X with $A\subseteq B$.Then $\lambda_{\beta c}Cl(A)\subseteq\lambda_{\beta c}Cl(B)$.

(7) For any subsets A,B of a space X. $\lambda_{\beta c}Cl(A)\cup\lambda_{\beta c}Cl(B)\subseteq\lambda_{\beta c}Cl(A\cup B)$.

(8) For any subsets A,B of a space X. $\lambda_{\beta c}Cl(A\cap B)\subseteq\lambda_{\beta c}Cl(A)\cap\lambda_{\beta c}Cl(B)$.

Proposition 2.15[3]. Let X be a space and $A\subseteq X$. Then $\lambda_{\beta c}Cl(A)=A\cup\lambda_{\beta c}D(A)$.

Definition 2.16.[4]. A space X is said to be λ-connected if there does not exist a pair A, B of nonempty disjoint λ-open subset of X such that $X = A\cup B$, otherwise X is called λ-disconnected. In this case, the pair (A, B) is called a λ-disconnection of X.

Definition 2.17.[5]. A space X is said to be λ_c-connected if there does not exist a pair A, B of nonempty disjoint λ_c-open subset of X such that $X = A \cup B$, otherwise X is called λ_c-disconnected. In this case, the pair (A, B) is called a λ_c-disconnection of X.

Theorem 2.17.[4]. Every s-connected space is λ-connected.

The converse of Theorem 2.17 is not true ingeneral by the following example:

Example 2.18.[4]. Let $X = \{a,b,c\}$, and $\tau = \{\phi, \{a\}, \{b\}, \{a,b\}, \{a,c\}, X\}$. We define an s-operation $\lambda : SO(X) \to P(X)$ as:

$$\lambda(A) = \begin{cases} X & \text{if } a \in A \\ A & \text{otherwise} \end{cases}.$$

$SO(X) = \{\phi, \{a\}, \{b\}, \{a,b\}, \{a,c\}, X\}$.

$SO_\lambda(X) = \{\phi, \{b\}, X\}$.

We have X is λ-connected, but it is not s-connected.

Theorem 2.19.[5]. Every λ-connected space is λ_c-connected.

Remark 2.20. We can show that, the converse of Theorem 2.19 is not true in general. In fact the space X of Example 2.6 is λ_c-connected, but not λ-connected.

Corollary 2.21.[5]. Every s-connected space is λ_c-connected.

3. $\lambda_{\beta c}$-Connected Spaces

In this section, we define and study some characterizations and properties of a new space called λ_{sc}-connected space.

We start this section with the following definitions.

Definition 3.1. Let X be a space and $Y \subseteq X$. Then the class of $\lambda_{\beta c}$-open sets in Y ($SO_{\lambda_{\beta c}}(Y)$) is defined in a natural way as:

$SO_{\lambda_{\beta c}}(Y) = \{Y \cap V : V \in SO_{\lambda_{\beta c}}(X)\}$.

That is W is $\lambda_{\beta c}$-open in Y if and only if $W = Y \cap V$, where V is a $\lambda_{\beta c}$-open set in X. Thus, Y is a subspace of X with respect to $\lambda_{\beta c}$-open set.

Definition 3.2. A space X is said to be $\lambda_{\beta c}$-connected if there does not exist a pair A, B of nonempty disjoint $\lambda_{\beta c}$-open subset of X such that $X = A \cup B$, otherwise X is called $\lambda_{\beta c}$-disconnected. In this case, the pair (A, B) is called a $\lambda_{\beta c}$- disconnection of X.

Definition 3.3. Let X be a space and $\lambda : SO(X) \to P(X)$ an s-operation, then the family $SO_{\lambda_{\beta c}}(X)$ is called $\lambda_{\beta c}$-indiscrete space if $SO_{\lambda_{\beta c}}(X) = \{\phi, X\}$.

Definition 3.4. Let X be a space and $\lambda : SO(X) \to P(X)$ an s-operation then the family $SO_{\lambda_{\beta c}}(X)$ is called a $\lambda_{\beta c}$-discrete space if $SO_{\lambda_{\beta c}}(X) = P(X)$.

Example 3.5. Every $\lambda_{\beta c}$-indiscrete space is λ_c-connected.

We give in below a characterization of $\lambda_{\beta c}$-connected spaces, the proof of which is straight forward.

Theorem 3.6. A space X is $\lambda_{\beta c}$-disconnected (respt. $\lambda_{\beta c}$-connected) if and only if there exists (resp., does not exist) a non empty proper subset A of X, which is both $\lambda_{\beta c}$-open and $\lambda_{\beta c}$-closed in X .

Theorem 3.7. Every λ-connected space is $\lambda_{\beta c}$-connected.

Proof. Let X be λ-connected, then there does not exist a pair A, B of nonempty disjoint λ-open subset of X such that $X = A \cup B$, but every $\lambda_{\beta c}$-open set is a λ-open set by Proposition 2.4, so there does not exist a pair A, B of nonempty disjoint $\lambda_{\beta c}$-open subset of X such that $X = A \cup B$. Thus X is $\lambda_{\beta c}$-connected.

The converse of Theorem 3.7 is not true in general as it is shown by the following example:

Example 3.8. Let $X=\{a,b,c\}$, and $\tau=\{\phi, \{a\}, \{b\}, \{a,b\}, \{a,c\}, X\}$. We define an s-operation $\lambda : SO(X)\rightarrow P(X)$ as follows:

$$\lambda(A)=\begin{cases} A & \text{if } A=\{a\} \\ X & \text{otherwise} \end{cases}.$$

$SO(X)=\{\phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, X\}=\beta O(X)$.

$SO_{\lambda}(X)=\{\phi, \{a\}, X\}$.

$SO_{\lambda_{\beta c}}(X)=\{\phi, X\}$.

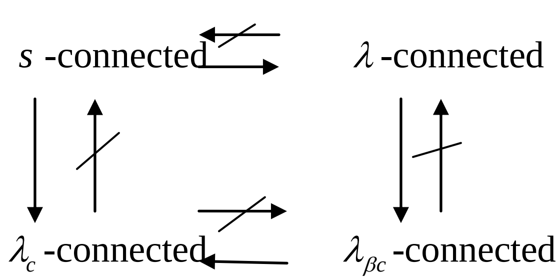
We have X is $\lambda_{\beta c}$-connected, but it is not λ-connected.

Theorem 3.9. Every $\lambda_{\beta c}$-connected space is λ_c-connected.

Proof. Let X be $\lambda_{\beta c}$-connected, then there does not exist a pair *A*, *B* of nonempty disjoint $\lambda_{\beta c}$-open subset of *X* such that *X* = *A* ∪ *B*, but every λ_c-open set is a $\lambda_{\beta c}$-open set by Proposition 2.4, so there does not exist a pair *A*, *B* of nonempty disjoint λ_c-open subset of *X* such that *X* = *A* ∪ *B*. Thus X is λ_c-connected.

Remark 3.10. The converse of Theorem 3.9 is not true, in general. The space X of Example 2.6 is λ_c-connected, but not $\lambda_{\beta c}$-connected.

Remark 3.11. The following diagram combining Theorem 2.17, Theorem 2.19, Theorem 3.7, Theorem 3.9, Corollary 2.21, Example 2.6, Example 2.18, Example 3.8, Remark 2.20 and Remark 3.10.



Definition 3.12. Let X be a space and $A \subseteq X$. The $\lambda_{\beta c}$ -boundary of A, written $\lambda_{\beta c}Bd(A)$, is defined as the set $\lambda_{\beta c}Bd(A) = \lambda_{\beta c}Cl(A) \cap \lambda_{\beta c}Cl(X / A)$.

Theorem 3.13. A space X is $\lambda_{\beta c}$-connected if and only if every nonempty proper subspace has a nonempty $\lambda_{\beta c}$-boundary.

Proof. Suppose that a nonempty proper subspace A of a $\lambda_{\beta c}$-connected space X has empty $\lambda_{\beta c}$-boundary. Then A is $\lambda_{\beta c}$-open and $\lambda_{\beta c}Cl(A) \cap \lambda_{\beta c}Cl(X \setminus A) = \phi$. Let p be a $\lambda_{\beta c}$-limit point of A. Then $p \in \lambda_{\beta c}Cl(A)$ but $p \notin \lambda_{\beta c}Cl(X \setminus A)$. In particular $p \notin (X \setminus A)$ and so $p \in A$. Thus A is $\lambda_{\beta c}$-closed and $\lambda_{\beta c}$-open. By Theorem 3.6, X is $\lambda_{\beta c}$-disconnected. This contradiction gives that A has a nonempty $\lambda_{\beta c}$-boundary.

Conversely, suppose X is $\lambda_{\beta c}$-disconnected. Then by Theorem 3.6, X has a proper subspace A which is both $\lambda_{\beta c}$ -closed and $\lambda_{\beta c}$ -open. Then $\lambda_{\beta c}Cl(A) = A$, $\lambda_{\beta c}Cl(X \setminus A) = (X \setminus A)$ and $\lambda_{\beta c}Cl(A) \cap \lambda_{\beta c}Cl(X \setminus A) = \phi$. So A has empty $\lambda_{\beta c}$ -boundary, a contradiction. Hence X is $\lambda_{\beta c}$-connected. This completes the proof.■

Theorem 3.14. Let (A, B) be a $\lambda_{\beta c}$-disconnection of a space X and C be a $\lambda_{\beta c}$-connected subspace of X. Then C is contained in A or in B.

Proof. Suppose that C is neither contained in A nor in B. Then $C \cap A$, $C \cap B$ are both nonempty $\lambda_{\beta c}$-open subsets of C such that $(C \cap A) \cap (C \cap B) = \phi$ and $(C \cap A) \cup (C \cap B) = C$. This gives that $(C \cap A, C \cap B)$ is a $\lambda_{\beta c}$-disconnection of C. This contradiction proves the theorem.■

Theorem 3.15. Let $X = \bigcup_{\alpha \in I} X_\alpha$, where each X_α is $\lambda_{\beta c}$-connected and $\bigcap_{\alpha \in I} X_\alpha \neq \phi$. Then X is $\lambda_{\beta c}$-connected.

Proof. Suppose on the contrary that (A, B) is a $\lambda_{\beta c}$-disconnection of X. Since each X_α is $\lambda_{\beta c}$-connected , therefore by Theorem 3.14, $X_\alpha \subseteq A$ or $X_\alpha \subseteq B$. Since

$\bigcap_{\alpha \in I} X_\alpha \neq \phi$, therefore all X_α are contained in A or in B. This gives that, if $X \subseteq A$, then $B = \phi$ or if $X \subseteq B$, then $A = \phi$. This contradiction proves that X is $\lambda_{\beta c}$-connected. Which completes the proof.■

Using Theorem 3.15, we give a characterization of $\lambda_{\beta c}$-connectedness as follows:

Theorem 3.16. A space X is $\lambda_{\beta c}$-connected if and only if for every pair of points x, y in X, there is a $\lambda_{\beta c}$-connected subset of X, which contains both x and y.

Proof. The necessity is immediate since the $\lambda_{\beta c}$-connected space itself contains these two points.

For the sufficiency, suppose that for any two points x, y; there is a $\lambda_{\beta c}$-connected subspace $C_{x,y}$ of X such that x, $y \in C_{x,y}$. Let $a \in X$ be a fixed point and $\{C_{a,x}, x \in X\}$ a class of all $\lambda_{\beta c}$-connected subsets of X, which contain a and $x \in X$. Then $X = \bigcup_{x \in X} C_{a,x}$ and $\bigcap_{x \in X} C_{a,x} \neq \phi$. Therefore, by Theorem 3.15, X is $\lambda_{\beta c}$-connected. This completes the proof.■

Theorem 3.17. Let C be a $\lambda_{\beta c}$-connected subset of a space X and $A \subseteq X$ such that $C \subseteq A \subseteq \lambda_{\beta c}Cl(C)$. Then A is $\lambda_{\beta c}$-connected.

Proof. It is sufficient to show that $\lambda_{\beta c}Cl(C)$ is $\lambda_{\beta c}$-connected. On the contrary, suppose that $\lambda_{\beta c}Cl(C)$ is $\lambda_{\beta c}$-disconnected. Then there exists a $\lambda_{\beta c}$-disconnection (H, K) of $\lambda_{\beta c}Cl(C)$. That is, $H \cap C$, $K \cap C$ are $\lambda_{\beta c}$-open sets in C such that $(H \cap C) \cap (K \cap C) = (H \cap K) \cap C = \phi$ and $(H \cap C) \cap (K \cap C) = (H \cap K) \cap C = C$. This gives that $(H \cap C, K \cap C)$ is a $\lambda_{\beta c}$-disconnection of C, a contradiction. This proves that $\lambda_{\beta c}Cl(C)$ is $\lambda_{\beta c}$-connected.■

4. $\lambda_{\beta c}$-component of a set

We introduce the following definition

Definition 4.1. A maximal $\lambda_{\beta c}$-connected subset of a space *X* is called a $\lambda_{\beta c}$-component of *X*. If *X* itself is $\lambda_{\beta c}$-connected, then *X* is the only $\lambda_{\beta c}$-component of *X*.

Next we study the properties of $\lambda_{\beta c}$-components of a space X :

Theorem 4.2. Let (X,τ) be a topological space. Then

(1) For each $x \in X$, there is exactly one $\lambda_{\beta c}$-component of *X* containing *x*.

(2) Each $\lambda_{\beta c}$-connected subset of *X* is contained in exactly one $\lambda_{\beta c}$-component of *X*.

(3) A $\lambda_{\beta c}$-connected subset of *X*, which is both $\lambda_{\beta c}$-open and $\lambda_{\beta c}$-closed is a $\lambda_{\beta c}$-component , if λ is s-regular.

(4) Every $\lambda_{\beta c}$-component of *X* is $\lambda_{\beta c}$-closed in *X*.

Proof:

(1) Let $x \in X$ and $\{C_\alpha : \alpha \in I\}$ be a class of all $\lambda_{\beta c}$-connected subsets of *X* containing *x*. Put C $= \bigcup_{\alpha \in I} C_\alpha$, then by Theorem 3.15, *C* is $\lambda_{\beta c}$-connected and $x \in X$. Suppose $C \subseteq C^*$, for some $\lambda_{\beta c}$-connected subset C^* of *X*. Then $x \in C^*$ and hence C^* is one of the C_α's and hence $C^* \subseteq C$. Consequently $C = C^*$. This proves that *C* is a $\lambda_{\beta c}$-component of *X*, which contains *x*.

(2) Let *A* be a $\lambda_{\beta c}$-connected subset of *X*, which is not a $\lambda_{\beta c}$-component of *X*. Suppose that C_1, C_2 are $\lambda_{\beta c}$-components of *X* such that $A \subseteq C_1$, $A \subseteq C_2$. Since $C_1 \cap C_2 = \phi$, $C_1 \cup C_2$ is another $\lambda_{\beta c}$-connected set which contains C_1 as well as C_2, this contradicts the fact that C_1 and C_2 are $\lambda_{\beta c}$-components. This proves that *A* is contained in exactly one $\lambda_{\beta c}$-component of *X*.

(3) Suppose that A is a $\lambda_{\beta c}$-connected subset of X which is both $\lambda_{\beta c}$-open and $\lambda_{\beta c}$-closed. By (2), A is contained in exactly one $\lambda_{\beta c}$-component C of X. If A is a proper subset of C, and since λ is s-regular, therefore $C = (C \cap A) \cup (C \cap (X \setminus A))$ is a $\lambda_{\beta c}$-disconnection of C, a contradiction. Thus, $A = C$.

(4) Suppose a $\lambda_{\beta c}$-component C of X is not $\lambda_{\beta c}$-closed. Then, by Theorem 3.17, $\lambda_{\beta c}Cl(A)$ is $\lambda_{\beta c}$-connected containing a $\lambda_{\beta c}$-component C of X. This implies $C = \lambda_{\beta c}Cl(A)$ and hence C is $\lambda_{\beta c}$-closed. This completes the proof.■

We introduce the following definition

Definition 4.3. A space X is said to be locally $\lambda_{\beta c}$-connected if for any point $x \in X$ and any $\lambda_{\beta c}$-open set U containing x, there is a $\lambda_{\beta c}$-connected $\lambda_{\beta c}$-open set V such that $x \in V \subseteq U$.

Theorem 4.4. A $\lambda_{\beta c}$-open subset of $\lambda_{\beta c}$-locally connected space is $\lambda_{\beta c}$-locally connected.

Proof. Let U be a $\lambda_{\beta c}$-open subset of a $\lambda_{\beta c}$-locally connected space X. Let $x \in U$ and V a $\lambda_{\beta c}$-open nbd of x in U. Then V is a $\lambda_{\beta c}$-open nbd of x in X. Since X is $\lambda_{\beta c}$-locally connected, therefore there exists a $\lambda_{\beta c}$-connected, $\lambda_{\beta c}$-open nbd W of x such that $x \in W \subseteq V$. So that W is also a $\lambda_{\beta c}$-connected $\lambda_{\beta c}$-open nbd x in U such that $x \in W \subseteq U \subseteq V$ or $x \in W \subseteq V$. This proves that U is $\lambda_{\beta c}$-locally connected.■

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