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$\lambda_{\beta c}$-OPEN SETS AND TOPOLOGICAL PROPERTIES

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Abstract

In this paper we introduce the concept of $\lambda_{\beta c}$-open sets in topological spaces and study topological properties of $\lambda_{\beta c}$-derived, $\lambda_{\beta c}$-closure and $\lambda_{\beta c}$-interior of a set using the concept of $\lambda_{\beta c}$-open sets.

1.Introduction

Throughout, X denote a topological spaces. Let A be a subset of X, then the closure and the interior of A are denoted by $Cl(A)$ and $Int(A)$ respectively. A subset A of a topological space (X,τ) is said to be semi open [1], if $A \subseteq Cl(Int(A))$. The complement of a semi open set is said to be semi closed [1].The family of all semi open (resp. semi closed) sets in a topological space (X,τ) is denoted by $SO(X,\tau)$ or $SO(X)$ (resp. $SC(X,\tau)$ or $SC(X)$). A subset A of a topological space (X,τ) is said to be β-open [2], if $A \subseteq Cl(Int(Cl(A)))$. The complement of a β-open set is said to be β-closed [2].The family of all β-open (resp. β-closed) sets in a topological space (X,τ) is denoted by $\beta O(X,\tau)$ or $\beta O(X)$ (resp. $\beta C(X,\tau)$ or $\beta C(X)$). We consider λ as a function defined on $SO(X)$ into $P(X)$ and $\lambda: SO(X) \rightarrow P(X)$ is called an s-operation if $V \subseteq \lambda(V)$ for each non-empty semi open set V [3],[4]. It is assumed that $\lambda(\phi) = \phi$ and $\lambda(X) = X$ for any s-operation λ **[3]**,[4]. Let $\lambda: SO(X) \rightarrow P(X)$ be an s-operation, then a subset A of X is called a λ^*-open set [5], if for each $x \in A$ there exists a semi open set U such that $x \in U$ and $\lambda(U) \subseteq A$.

The complement of a λ^*-open set is said to be λ^*-closed set which is equivalent to λ-closed set[6]. The family of all λ^*-open (resp., λ^*-closed) subsets of a topological space (X,τ) is denoted by $SO_\lambda(X,\tau)$ or $SO_\lambda(X)$ (resp. $SC_\lambda(X,\tau)$ or $SC_\lambda(X)$). Let A be a subset of X. Then:

(1) The λ-closure of A ($\lambda^* Cl(A) = \lambda Cl(A)$) is the intersection of all λ^*-closed (λ-closed) sets containing A [5].

(2) The λ-interior of A ($\lambda^* Int(A) = \lambda Int(A)$) is the union of all λ^*-open(λ-open) sets of X contained in A [5].

(3) A point $x \in X$ is said to be a λ-limit point of *A* if every λ^*-open (λ-open)set containing *x* contains a point of *A* different from *x*, and the set of all λ-limit points of *A* is called the λ-derived set of *A* denoted by $\lambda^* d(A)$ ($\lambda d(A)$ [3] ,[4]).

An s-operation $\lambda : SO(X) \to P(X)$ is said to be:

(1) λ-identity on $SO(X)$ [7], if $\lambda(A) = A$ for all $A \in SO(X)$.

(2) λ-monotone on $SO(X)$ [7], if $A \subseteq B$ implies $\lambda(A) \subseteq \lambda(B)$ for all A, $B \in SO(X)$.

(3) λ-idempotent on $SO(X)$ [7], if $\lambda(\lambda(A)) = \lambda(A)$ for all $A \in SO(X)$.

(4) λ-additive on $SO(X)$ [7], if $\lambda(A \cup B) = \lambda(A) \cup \lambda(B)$, for all A, $B \in SO(X)$. If $\bigcup_{i \in I} \lambda(A_i) \subseteq \lambda(\bigcup_{i \in I} A_i)$ for any collection $\{A_i\}_{i \in I} \subseteq SO(X)$ then λ is said to be λ-sub additive [7], on $SO(X)$.

Definition 1.1 [6]

Let (X,τ) be a topological space. A s-operation λ is said to be λ-regular if for every semi open sets U and V of each $x \in X$, there exists a semi open set W of X

such that $\lambda(W) \subseteq \lambda(U) \cap \lambda(V)$. An s-operation λ is said to be λ-open if for every semi open set U containing $x \in X$, there exists a λ^*-open set V such that $x \in V$ and $V \subseteq \lambda(U)$.

Proposition 1.2 [8]

Let $\{F_\alpha\}_{\alpha \in J}$ be any collection of semi closed sets in a topological space (X, τ) then $\bigcap_{\alpha \in J} F_\alpha$ is a semi closed set.

Proposition 1.3 [3],[4]

For each point $x \in X$, $x \in \lambda^* Cl(A) = \lambda Cl(A)$ if and only if $V \cap A \neq \phi$, for every $V \in SO_\lambda(X)$ such that $x \in V$.

Proposition 1.4 [6]

For a topological space (X, τ), $SO_\lambda(X) \subseteq SO(X)$.

Proposition 1.5 [3],[4]

Let $\{A_\alpha\}_{\alpha \in I}$ be any collection of λ^*-open sets in a topological space (X, τ) then $\bigcup_{\alpha \in I} A_\alpha$ is a λ^*-open set.

Proposition 1.6 [6]

For a topological space (X, τ), $SC_\lambda(X) \subseteq SC(X)$.

2. $\lambda_{\beta c}$-open sets

In this section, we introduce a new class of semi open sets called $\lambda_{\beta c}$-open sets in topological spaces.

Definition 2.1

A λ^*-open subset A of a topological space (X, τ) is called $\lambda_{\beta c}$-open if for each $x \in A$ there exists a β-closed set K such that $x \in K \subseteq A$. The complement of a $\lambda_{\beta c}$-

open set is said to be $\lambda_{\beta c}$-closed. The family of all $\lambda_{\beta c}$-open (resp. $\lambda_{\beta c}$-closed) subsets of a topological space (X,τ) is denoted by $SO_{\lambda_{\beta c}}(X,\tau)$ or $SO_{\lambda_{\beta c}}(X)$ (resp. $SC_{\lambda_{\beta c}}(X,\tau)$ or $SC_{\lambda_{\beta c}}(X)$).

Proposition 2.2

For a topological space (X,τ), $SO_{\lambda_{\beta c}}(X) \subseteq SO_{\lambda}(X) \subseteq SO(X)$.

Proof. Every $\lambda_{\beta c}$-open set is λ-open set by Definition 2.1. And every λ^*-open set is semi open set by Proposition 1.4. This implies that $SO_{\lambda_{\beta c}}(X) \subseteq SO_{\lambda}(X) \subseteq SO(X)$.

Example 2.3

Let $X=\{a,b,c\}$, and $\tau=\{\phi,\{c\},\{a,c\},X\}$. We define an s-operation $\lambda : SO(X) \to P(X)$ as:

$$\lambda(A)=\begin{cases} A & \text{if} \quad A=\{a,c\} \text{ or } \phi \\ X & \text{Otherwise} \end{cases}.$$

$SO(X)=\{\phi,\{c\},\{a,c\},\{b,c\},X\}=\beta O(X)$

$SC(X)=\{\phi,\{a\},\{b\},\{a,b\},X\}=\beta C(X)$

$SO_{\lambda}(X)=\{\phi,\{a,c\},X\}$

$SO_{\lambda_{\beta c}}(X)=\{\phi,\{c\},X\}$

We have $\{a,c\} \in SO_{\lambda}(X)$ but $\{a,c\} \notin SO_{\lambda_{\beta c}}(X)$.

Example 2.4

Let $X=\{a,b,c\}$, and $\tau=\{\phi,\{a\},\{c\},\{a,c\},X\}$. We define an s-operation $\lambda : SO(X) \to P(X)$ as:

$$\lambda(A)=\begin{cases} A & \text{if} \quad A=\{a,b\} \text{ or } \{b,c\} \text{ or } \phi \\ X & \text{Otherwise} \end{cases}.$$

$SO(X)=\{\phi,\{a\},\{c\},\{a,b\},\{a,c\},\{b,c\},X\}=\beta O(X)$.

$SC(X) = \{\phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, X\} = \beta C(X)$.

$SO_{\lambda}(X) = \{\phi, \{a,b\}, \{b,c\}, X\}$

$SO_{\lambda_{\beta c}}(X) = \{\phi, \{a,b\}, \{b,c\}, X\}$.

We have $\{b,c\} \in SO_{\lambda_{\beta c}}(X)$ but $\{b,c\} \notin \tau$ and $\{a\} \in \tau$ but $\{a\} \notin SO_{\lambda_{\beta c}}(X)$.

Thus the family open sets and $\lambda_{\beta c}$-open set are independent.

The following result shows that any union of $\lambda_{\beta c}$-open set in a topological space (X, τ) is $\lambda_{\beta c}$-open set.

Proposition 2.5

Let $\{A_\alpha\}_{\alpha \in I}$ be any collection of $\lambda_{\beta c}$-open sets in a topological space (X, τ) then $\bigcup_{\alpha \in I} A_\alpha$ is a $\lambda_{\beta c}$-open set.

Proof. Let $x \in \bigcup_{\alpha \in I} A_\alpha$. Then there exist $\alpha_0 \in I$ such that $x \in A_{\alpha 0}$. Since A_α is a $\lambda_{\beta c}$-open set for all $\alpha \in I$ then A_α is a λ^*-open set for all $\alpha \in I$. This implies that there exists a semi open set U such that $\lambda(U) \subseteq A_{\alpha_0} \subseteq \bigcup_{\alpha \in I} A_\alpha$ therefore $\bigcup_{\alpha \in I} A_\alpha$ is a λ^*-open subset of (X, τ). Let $x \in \bigcup_{\alpha \in I} A_\alpha$ there exist $\beta \in I$ such that $x \in A_\beta$. Since A_α is a $\lambda_{\beta c}$-open set for all $\alpha \in I$, then there exist a β-closed set K such that $x \in K \subseteq A_\beta$ but $A_\beta \subseteq \bigcup_{\alpha \in I} A_\alpha$ then $x \in K \subseteq \bigcup_{\alpha \in I} A_\alpha$. Hence $\bigcup_{\alpha \in I} A_\alpha$ is a $\lambda_{\beta c}$-open set.

The following example shows that the intersection of two $\lambda_{\beta c}$-open sets need not be $\lambda_{\beta c}$-open.

Example 2.6

Let $X=\{a,b,c\}$, and $\tau=P(X)$. We define an s-operation $\lambda: SO(X)\rightarrow P(X)$ as:

$$\lambda(A)=\begin{cases} A & \text{if} \quad A=\{a,b\} \text{ or } \{b,c\} \text{ or } \phi \\ X & \text{Otherwise} \end{cases}.$$

$SO(X)=P(X)=\beta O(X)$.

$SC(X)=P(X)=\beta C(X)$.

$SO_{\lambda}(X)=\{\phi,\{a,b\},\{b,c\},X\}$

$SO_{\lambda_{\beta c}}(X)=\{\phi,\{a,b\},\{b,c\},X\}$.

We have $\{a,b\}$ and $\{b,c\}$ are $\lambda_{\beta c}$-open sets but $\{a,b\}\cap\{b,c\}=\{b\}$ is not $\lambda_{\beta c}$-open.

Proposition 2.7

The set A is $\lambda_{\beta c}$-open set in the topological space (X,τ) if and only if for each $x\in A$ there exists a $\lambda_{\beta c}$-open set B such that $x\in B\subseteq A$.

Proof. Suppose that A is $\lambda_{\beta c}$-open set in the topological space (X,τ). Then for each $x\in A$, put $B=A$ is a $\lambda_{\beta c}$-open such that $x\in B\subseteq A$.

Conversely, suppose that for each $x\in A$ there exists a $\lambda_{\beta c}$-open set B such that $x\in B_x\subseteq A$, thus $A=\bigcup B_x$, where $B_x\in SO_{\lambda_{\beta c}}(X)$ for each x. Therefore, A is a $\lambda_{\beta c}$-open set by Proposition 2.5.

Proposition 2.8

If the family of all semi open sets of a space X is a topology on X and λ is a λ-

monotone s-operation, then the family of $\lambda_{\beta c}$-open sets is also a topology on X.

Proof. Clearly $\phi, X \in SO_{\lambda_{\beta c}}(X)$ and by Proposition 2.5 the union of any family of $\lambda_{\beta c}$-open sets is $\lambda_{\beta c}$-open. To complete the proof it is enough to show the finite intersection of $\lambda_{\beta c}$-open sets is $\lambda_{\beta c}$-open. Let A and B be two $\lambda_{\beta c}$-open sets. Then A and B are both λ^*-open and semi open sets. Since $SO(X)$ is a topology on X, so $A \cap B$ is semi open. Let $x \in A \cap B$ then $x \in A$ and $x \in B$, then there exist semi open sets F and E such that $x \in F \subseteq \lambda(F) \subseteq A$, and $x \in E \subseteq \lambda(E) \subseteq B$, since λ is a λ-monotone s-operation and $F \cap E$ is semi open set such that $F \cap E \subseteq F$ and $F \cap E \subseteq E$, this implies that $\lambda(F \cap E) \subseteq \lambda(F) \cap \lambda(E) \subseteq A \cap B$. Thus $A \cap B$ is λ^*-open set. Let $x \in A \cap B$. Then $x \in A$ and $x \in B$, but A and B are $\lambda_{\beta c}$-open sets, so there exist β-closed sets K_1 and K_2 such that $x \in K_1 \subseteq A$ and $x \in K_2 \subseteq B$ which implies that $x \in K_1 \cap K_2 \subseteq A \cap B$ then $x \in K \subseteq A \cap B$, where $K = K_1 \cap K_2$, but $K_1 \cap K_2$ is β-closed set by Proposition 1.2, then $A \cap B$ is a $\lambda_{\beta c}$-open set. Hence $A \cap B \in SO_{\lambda_{\beta c}}(X)$. Thus the family of $\lambda_{\beta c}$-open sets form a topology on X.

Proposition 2.9

Let $\{K_\alpha\}_{\alpha \in J}$ be any collection of $\lambda_{\beta c}$-closed sets in a topological space (X, τ) then $\bigcap_{\alpha \in J} K_\alpha$ is a $\lambda_{\beta c}$-closed set.

Proof. Obvious

Proposition 2.10

For a topological space (X, τ), $SC_{\lambda_{\beta c}}(X) \subseteq SC_\lambda(X) \subseteq SC(X)$.

Proof. Obvious

Theorem 2.11

Let λ be λ-regular s-operation. If A and B are $\lambda_{\beta c}$-open sets in X, then $A \cap B$ is also a $\lambda_{\beta c}$-open set.

Proof. Let $x \in A \cap B$. Then $x \in A$ and $x \in B$. Since A and B are $\lambda_{\beta c}$-open sets, there exists semi open sets U and V such that $x \in U$ and $\lambda(U) \subseteq A$, $x \in V$ and $\lambda(V) \subseteq B$. Since λ is a λ-regular s-operation, this implies there exists a semi open set W of X such that $\lambda(W) \subseteq \lambda(U) \cap \lambda(V) \subseteq A \cap B$. This implies that $A \cap B$ is λ^*-open set. Let $x \in A \cap B$. Then $x \in A$ and $x \in B$, since A and B are $\lambda_{\beta c}$-open sets then there exist β-closed sets K_1 and K_2 such that $x \in K_1 \subseteq A$ and $x \in K_2 \subseteq B$, then $x \in K_1 \cap K_2 \subseteq A \cap B$. Since $K_1 \cap K_2$ is β-closed set. Thus $A \cap B$ is a $\lambda_{\beta c}$-open set.

3.1 Some properties of $\lambda_{\beta c}$-open sets

In the present section we study topological properties of $\lambda_{\beta c}$-derived, $\lambda_{\beta c}$-closure and $\lambda_{\beta c}$-interior using the concept of $\lambda_{\beta c}$-open sets.

Definition 3.1

Let A be a subset of a space X. A point $x \in X$ is said to be a $\lambda_{\beta c}$-limit point of A if for each $\lambda_{\beta c}$-open set U containing x, then $U \cap (A \setminus \{x\}) \neq \phi$. The set of all $\lambda_{\beta c}$-limit points of A is called a $\lambda_{\beta c}$-derived set of A and is denoted by $\lambda_{\beta c}D(A)$.

Lemma 3.2

Let A and B be subsets of a space X. If $A \subseteq B$ then $\lambda_{\beta c}D(A) \subseteq \lambda_{\beta c}D(B)$. **Proof.** Obvious.

But in general $\lambda_{\beta c}D(A) = \lambda_{\beta c}D(B)$ does not imply $A = B$. For this we give the following example:

Example 3.3

Let $X=\{a,b,c\}$, and $\tau=\{\phi,\{b\},\{b,c\},X\}$. We define an s-operation $\lambda:SO(X)\to P(X)$ as:

$$\lambda(A)=\begin{cases} A & \text{if } A=\{a,b\} \text{ or } \{b,c\} \text{ or } \phi \\ X & \text{Otherwise} \end{cases}.$$

$SO(X)=\{\phi,\{b\},\{a,b\},\{b,c\},X\}=\beta O(X)$,

$SC(X)=\{\phi,\{c\},\{a\},\{a,c\},X\}=\beta C(X)$.

$SO_{\lambda}(X)=\{\phi,\{a,b\},\{b,c\},X\}$.

$SO_{\lambda_{\beta c}}(X)=\{\phi,X\}$. If $A=\{a,c\}$ and $B=\{b,c\}$, then $\lambda_{\beta c}D(A)=\lambda_{\beta c}D(B)=X$, but $A\neq B$.

Some properties of $\lambda_{\beta c}$-derived sets are stated in the following proposition.

Proposition 3.4

Let *A*, *B* be any two subsets of a space X and $\lambda:SO(X)\to P(X)$ be an s-operation. Then we have the following properties:

(1) $\lambda_{\beta c}D(\phi)=\phi$.

(2) If $x\in\lambda_{\beta c}D(A)$, then $x\in\lambda_{\beta c}D(A\setminus\{x\})$.

(3) $\lambda_{\beta c}D(A)\cup\lambda_{\beta c}D(B)\subseteq\lambda_{\beta c}D(A\cup B)$.

(4) $\lambda_{\beta c}D(A\cap B)\subseteq\lambda_{\beta c}D(A)\cap\lambda_{\beta c}D(B)$.

(5) $\lambda_{\beta c}D(\lambda_{\beta c}D(A))\setminus A\subseteq\lambda_{\beta c}D(A)$.

(6) $\lambda_{\beta c}D(A\cup\lambda_{\beta c}D(A))\subseteq A\cup\lambda_{\beta c}D(A)$.

Proof.(1) Let $x \in X$ be arbitrary point of X , and let U be any $\lambda_{\beta c}$-open set which contains x such that $(\phi \setminus \{x\} \cap U) = \phi$ then $x \notin \lambda_{\beta c}D(\phi)$. There fore $\lambda_{\beta c}D(\phi) = \phi$.

(2) Let $x \in X$ and $x \in \lambda_{\beta c}D(A)$, then by Definition 3.1, for any $U \in SO_{\lambda_{\beta c}}(X)$, we have $U \cap (A \setminus \{x\}) \neq \phi$, but $(A \setminus \{x\}) = (A \setminus \{x\}) \setminus \{x\}$. Thus $((A \setminus \{x\}) \setminus \{x\}) \cap U \neq \phi$. Therefore $x \in \lambda_{\beta c}(A \setminus \{x\})$.

(3) We have $A \subseteq A \cup B$ and $B \subseteq A \cup B$ generally, then $\lambda_{\beta c}D(A) \subseteq \lambda_{\beta c}D(A \cup B)$ and $\lambda_{\beta c}D(B) \subseteq \lambda_{\beta c}D(A \cup B)$, therefore $\lambda_{\beta c}D(A) \cup \lambda_{\beta c}D(B) \subseteq \lambda_{\beta c}D(A \cup B)$.

(4) We have $A \cap B \subseteq A$ and $A \cap B \subseteq B$ generally, then $\lambda_{\beta c}D(A \cap B) \subseteq \lambda_{\beta c}D(A)$ and $\lambda_{\beta c}D(A \cap B) \subseteq \lambda_{\beta c}D(B)$, therefore $\lambda_{\beta c}D(A \cap B) \subseteq \lambda_{\beta c}D(A) \cap \lambda_{\beta c}D(B)$.

(5) If $x \in (\lambda_{\beta c}D(\lambda_{\beta c}D(A)) \setminus A)$ and U is a $\lambda_{\beta c}$-open set containing x then $U \cap (\lambda_{\beta c}D(A) \setminus \{x\}) \neq \phi$. Let $y \in U \cap (\lambda_{\beta c}D(A) \setminus \{x\})$ then since $y \in \lambda_{\beta c}D(A)$ and $y \in U$, so $z \in U \cap (A \setminus \{y\}) \neq \phi$. Let Then $z \neq x$ for $z \in A$ and $x \notin A$, implies that $U \cap (A \setminus \{x\}) \neq \phi$. Therefore $x \in \lambda_{\beta c}D(A)$.

(6) Let $x \in \lambda_{\beta c}D(A \cup \lambda_{\beta c}D(A))$. If $x \in A$, the result is obvious. So, let $x \in (\lambda_{\beta c}D(A \cup \lambda_{\beta c}D(A)) \setminus A)$. Then, for any $\lambda_{\beta c}$-open sets U containing x, $U \cap ((A \cup \lambda_{\beta c}D(A)) \setminus \{x\}) \neq \phi$. Thus $U \cap (A \setminus \{x\}) \neq \phi$ or $U \cap (\lambda_{\beta c}D(A) \setminus \{x\}) \neq \phi$. Now, it follows similarly from(5) that $U \cap (A \setminus \{x\}) \neq \phi$. Hence $x \in \lambda_{\beta c}D(A)$. Therefore in any case, $\lambda_{\beta c}D(A \cup \lambda_{\beta c}D(A)) \subseteq A \cup \lambda_{\beta c}D(A)$.

In general the equalities of (3),(4) and (5) of the above proposition does not hold; it is shown in the following examples.

Example 3.5

Let $X=\{a,b,c,d\}$, and $\tau = P(X)$. We define an s-operation $\lambda : SO(X) \to P(X)$ as:

$$\lambda(A)=\begin{cases} A & \text{if } A=\{a,c\} \text{ or } \{b,c\} \text{ or } \phi \\ X & \text{Otherwise} \end{cases}.$$

$SO(X)=SC(X)=P(X)=\beta O(X)=\beta C(X)$.

$SO_{\lambda}(X)=\{\phi,\{a,c\},\{b,c\}X\}$.

$SO_{\lambda_{\beta c}}(X)=\{\phi,\{a,c\},\{b,c\}X\}$.

Let $A=\{a,c\}$ and $B=\{b,d\}$. then $\lambda_{\beta c}D(A)=\{a,b,d\}$, $\lambda_{\beta c}D(B)=\{a,b,d\}$.

$\lambda_{\beta c}D(A)\cup\lambda_{\beta c}D(B)=\{a,b,d\}$. But $A\cup B=X$, so $\lambda_{sc}D(A\cup B)=X$.

Hence $\lambda_{\beta c}D(A)\cup\lambda_{\beta c}D(B)\neq\lambda_{\beta c}D(A\cup B)$.

Example 3.6

Let $X=\{a,b,c,d\}$, and $\tau = P(X)$. We define an s-operation $\lambda : SO(X) \to P(X)$ as:

$$\lambda(A)=\begin{cases} A & \text{if } A=\{a,b,d\} \text{ or } \phi \\ X & \text{Otherwise} \end{cases}.$$

$SO(X)=P(X)=\beta O(X)=\beta C(X)=\tau$,

$SO_{\lambda}(X)=\{\phi,\{a,b,d\},X\}$,

$SO_{\lambda_{\beta c}}(X)=\{\phi,\{a,b,d\},X\}$.

Let $A=\{a,c\}$ and $B=\{b,c\}$. Then $\lambda_{\beta c}D(A)=\{b,c,d\}$, $\lambda_{\beta c}D(B)=\{a,c,d\}$

$\lambda_{\beta c}D(A)\cap\lambda_{\beta c}D(B)=\{b,c,d\}\cap\{a,c,d\}=\{c,d\}$ and $A\cap B=\{a,c\}\cap\{b,c\}=\{c\}$.

Then $\lambda_{\beta c}D(A\cap B)=\phi$, where Hence $\lambda_{\beta c}D(A)\cap\lambda_{\beta c}D(B)\neq \lambda_{\beta c}D(A\cap B)$. We see $\lambda_{\beta c}D(\lambda_{\beta c}D(A))=\lambda_{\beta c}D(\{b,c\})=\{a,b\}$. Then $\lambda_{\beta c}D(\lambda_{\beta c}D(A))\setminus A=\phi$, but $\lambda_{\beta c}D(A)=\{b,c\}$. This implies that $\lambda_{\beta c}D(\lambda_{\beta c}D(A))\setminus A\neq\lambda_{\beta c}D(A)$.

Proposition 3.7

Let (X,τ) be a topological space and $A\subseteq X$. Then A is a $\lambda_{\beta c}$-closed subset of X if and only if $\lambda_{\beta c}D(A)\subseteq A$.

Proof. Let A be a $\lambda_{\beta c}$-closed subset of X. Let $x\in\lambda_{\beta c}D(A)$ then $x\in A$ or $x\in X\setminus A$ if $x\in A$ there is nothing to prove, but if $x\in X\setminus A$ then $x\in X\setminus A$, $X\setminus A$ is a $\lambda_{\beta c}$-open, $X\setminus A\cap(A\setminus\{x\})=\phi$ then $x\notin\lambda_{\beta c}D(A)$, we get contradiction . Hence $\lambda_{\beta c}D(A)\subseteq A$.

Conversely: Let if $\lambda_{\beta c}D(A)\subseteq A$ then $x\notin A$, then $x\notin\lambda_{\beta c}D(A)$ so there exist a $\lambda_{\beta c}$-open set G which contain x such that $G\cap(A\setminus\{x\})=\phi$ but $x\notin A$, so $G\cap A=\phi$, then $G\subseteq X\setminus A$, then $x\in G\subseteq X\setminus A$ since G is $\lambda_{\beta c}$-open set, then $X\setminus A$ is $\lambda_{\beta c}$-open set by Proposition 2.7, then A is $\lambda_{\beta c}$-closed set.

The following example shows that the $\lambda_{\beta c}$-derived set is not $\lambda_{\beta c}$-closed set in general.

Example 3.8

Let $X=\{a,b,c\}$, and $\tau=\{\phi,\{a\},\{b\},\{a,b\},X\}$. We define an s-operation $\lambda:SO(X)\rightarrow P(X)$ as:

$$\lambda(A)=\begin{cases}A & \text{if } A=\{a,c\}\ or\ \{b,c\}\ or\ \phi\\ X & \text{Otherwise}\end{cases}.$$

$SO(X)=\{\phi,\{a\},\{b\},\{a,b\},\{b,c\},\{a,c\},X\}=\beta O(X).$

$SC(X)=\{\phi,\{a\},\{b\},\{c\},\{b,c\},\{a,c\},X\}=\beta C(X).$

$SO_{\lambda}(X)=\{\phi,\{a,c\},\{b,c\},X\}.$

$SO_{\lambda_{\beta c}}(X)=\{\phi,\{a,c\},\{b,c\},X\}$. Now, if we let $A=\{a,c\}$, then $\lambda_{\beta c}D(A)=\{a,b\}$, but $\lambda_{\beta c}D(A)$ is not $\lambda_{\beta c}$-closed set.

Corollary 3.9

Let (X,τ) be a topological space and $A\subseteq X$. Then $\lambda^{*}d(A)\subseteq\lambda_{\beta c}D(A)$.

Proof. Let $x\in\lambda^{*}d(A)$ then $(A\setminus\{x\})\cap U\neq\phi$, for every $U\in SO_{\lambda}(X)$, then $(A\setminus\{x\})\cap U\neq\phi$, for every $U\in SO_{\lambda_{\beta c}}(X)$, then $x\in\lambda_{\beta c}D(A)$. Hence $\lambda^{*}d(A)\subseteq\lambda_{\beta c}D(A)$. Finally, we have $\lambda^{*}d(A)\subseteq\lambda_{\beta c}D(A)$.

The converse of the above proposition is not true in general as shown by following example.

Remark 3.10

The converse of the Corollary 3.9 is not true in general. Now, in Example 3.3, if we let $A=\{b,c\}$ then $\lambda_{\beta c}D(A)=X$, $\lambda^{*}d(A)=\{a,c\}$, but $\lambda_{\beta c}D(A)\not\subseteq\lambda^{*}d(A)$.

Definition 3.11

For any subset A of a topological space (X,τ), the $\lambda_{\beta c}$-closure of A, denoted by $\lambda_{\beta c}Cl(A)$, is the intersection of all $\lambda_{\beta c}$-closed sets containing A.

Here we introduce some properties of $\lambda_{\beta c}$-closure of the sets.

Proposition 3.12

For subsets A, B of a topological space (X,τ), the following statements are true.

(1) $A \subseteq \lambda_{\beta c}Cl(A)$.

(2) $\lambda_{\beta c}Cl(A)$ is $\lambda_{\beta c}$ -closed set in X.

(3) $\lambda_{\beta c}Cl(A)$ is smallest $\lambda_{\beta c}$-closed set which contain A.

(4) A is $\lambda_{\beta c}$-closed set if and only if $A = \lambda_{\beta c}Cl(A)$.

(5) $\lambda_{\beta c}Cl(\phi) = \phi$ and $\lambda_{\beta c}Cl(X) = X$.

(6) If $\lambda_{\beta c}Cl(A) \cap \lambda_{\beta c}Cl(B) = \phi$, then $A \cap B = \phi$.

(7) $\lambda_{\beta c}Cl(A) = A \cup \lambda_{\beta c}D(A)$.

(8) If $A \subseteq B$. Then $\lambda_{\beta c}Cl(A) \subseteq \lambda_{\beta c}Cl(B)$.

(9) $\lambda_{\beta c}Cl(A) \cup \lambda_{\beta c}Cl(B) \subseteq \lambda_{\beta c}Cl(A \cup B)$.

(10) $\lambda_{\beta c}Cl(A \cap B) \subseteq \lambda_{\beta c}Cl(A) \cap \lambda_{\beta c}Cl(B)$.

Proof. (1) From the definition $\lambda_{\beta c}Cl(A) = \bigcap_{A \subseteq F} F$, where F is $\lambda_{\beta c}$-closed set, then $A \subseteq \lambda_{\beta c}Cl(A)$.

(2) $\lambda_{\beta c}Cl(A) = \bigcap_{A \subseteq F} F$, where F is $\lambda_{\beta c}$-closed set, then $\bigcap_{A \subseteq F} F$ is $\lambda_{\beta c}$-closed set by Proposition 2.9. Then $\lambda_{\beta c}Cl(A)$ is $\lambda_{\beta c}$-closed set in X. From (1) and (2) we get $\lambda_{\beta c}Cl(A)$ is $\lambda_{\beta c}$-closed and contain A, it is enough to show $\lambda_{\beta c}Cl(A)$ is smallest.

(3) Let H be any $\lambda_{\beta c}$-closed set such that $A \subseteq H$, then $\bigcap_{A \subseteq F} F \subseteq H$ then $\lambda_{\beta c}Cl(A) \subseteq H$. Therefore $\lambda_{\beta c}Cl(A)$ is smallest $\lambda_{\beta c}$-closed set which contain A.

(4) Let A be a $\lambda_{\beta c}$-closed set, then A is smallest $\lambda_{\beta c}$-closed set which contain A. Therefore $\lambda_{\beta c}Cl(A) = A$. Conversely, Let $A = \lambda_{\beta c}Cl(A)$ then A is $\lambda_{\beta c}$-closed set.

(5) Since ϕ and X are $\lambda_{\beta c}$-closed sets. Therefore $\lambda_{\beta c}Cl(\phi) = \phi$ and $\lambda_{\beta c}Cl(X) = X$.

(6) If possible suppose that $A \cap B \neq \phi$, there exist $x \in X$ and $x \in A \cap B$, then $x \in A$ and $x \in B$, therefore $x \in \lambda_{\beta c}Cl(A)$ and $x \in \lambda_{\beta c}Cl(B)$. Then $x \in \lambda_{\beta c}Cl(A) \cap \lambda_{\beta c}Cl(B)$, a contradiction. Therefore $A \cap B = \phi$.

(7) Since $\lambda_{\beta c}D(A) \subseteq \lambda_{\beta c}Cl(A)$ and $A \subseteq \lambda_{\beta c}Cl(A)$ then $A \cup \lambda_{\beta c}D(A)$ $\subseteq \lambda_{\beta c}Cl(A)$. On the other hand. To show $\lambda_{\beta c}Cl(A) \subseteq A \cup \lambda_{\beta c}D(A)$ since $\lambda_{\beta c}Cl(A)$ is the smallest $\lambda_{\beta c}$-closed set containing A so it is enough to prove that $A \cup \lambda_{\beta c}D(A)$ is $\lambda_{\beta c}$-closed set. Let $x \notin A \cup \lambda_{\beta c}D(A)$. Implies that $x \notin A$ and $x \notin \lambda_{\beta c}Cl(A)$. Since $x \notin \lambda_{\beta c}D(A)$, there exists a $\lambda_{\beta c}$-open set G of x which contains no point of A other than x but $x \notin A$. So G contains no point of A which this implies that $G \subseteq X \setminus A$. Again, G is a $\lambda_{\beta c}$-open set of each of its points. But as G does not contain any point of A, no point of G can be a $\lambda_{\beta c}$-limit point of A. Therefore, no point of G can belong to $\lambda_{\beta c}D(A)$. This implies that $G \subseteq X \setminus \lambda_{\beta c}D(A)$. Hence it follows that $x \in G \subseteq$ $X \setminus A \cap X \setminus \lambda_{\beta c}D(A) = X \setminus (A \cup \lambda_{\beta c}D(A))$, therefore, $A \cup \lambda_{\beta c}D(A)$ is $\lambda_{\beta c}$-closed set. Hence $\lambda_{\beta c}Cl(A) \subseteq A \cup \lambda_{\beta c}D(A)$. Thus $\lambda_{\beta c}Cl(A) = A \cup \lambda_{\beta c}D(A)$.

(8) We have $A \subseteq B$ this implies that $\lambda_{\beta c}D(A) \subseteq \lambda_{\beta c}D(B)$ by Lemma 3.2.

Therefore $A \cup \lambda_{\beta c}D(A) \subseteq B \cup \lambda_{\beta c}D(B),$ since $A \subseteq B,$ this implies that

$\lambda_{\beta c}Cl(A) \subseteq \lambda_{\beta c}Cl(B)$ by (7).

(9) Since $A \subseteq A \cup B$ and $B \subseteq A \cup B$, this implies that $\lambda_{\beta c}Cl(A) \subseteq$

$\lambda_{\beta c}Cl(A \cup B)$ and $\lambda_{\beta c}Cl(B) \subseteq \lambda_{\beta c}Cl(A \cup B)$, by (8). So

$\lambda_{\beta c}Cl(A) \cup \lambda_{\beta c}Cl(B) \subseteq \lambda_{\beta c}Cl(A \cup B)$.

(10) Since $A \cap B \subseteq A$ and $A \cap B \subseteq B$ this implies that $\lambda_{\beta c}Cl(A \cap B) \subseteq$

$\lambda_{\beta c}Cl(A)$ and $\lambda_{\beta c}Cl(A \cap B) \subseteq \lambda_{\beta c}Cl(B)$, by (8). So

$\lambda_{\beta c}Cl(A \cap B) \subseteq \lambda_{\beta c}Cl(A) \cap \lambda_{\beta c}Cl(B)$.

Proposition 3.13

For each point $x \in X$, $x \in \lambda_{\beta c}Cl(A)$ if and only if $V \cap A \neq \phi$, for every $V \in SO_{\lambda_{\beta c}}(X)$ such that $x \in V$.

Proof. Let $x \in \lambda_{\beta c}Cl(A)$ and suppose that $V \cap A \neq \phi$, for some $\lambda_{\beta c}$-open set V which contains x. Then $X \setminus V$ is $\lambda_{\beta c}$-closed and $A \subseteq (X \setminus V)$, thus $\lambda_{\beta c}Cl(A) \subseteq (X \setminus V)$. Implies that $x \in (X \setminus V)$, a contradiction. Therefore $V \cap A \neq \phi$.

The converse of Proposition 3.12(8) is not true in general as it is shown by the following example:

Remark 3.14

In Example 3.3 if we let $A = \{a, c\}$ and $B = \{b, c\}$ then $\lambda_{\beta c}Cl(A) = \lambda_{\beta c}Cl(B) = X$, but $A \neq B$.

In general the equalities of the Proposition 3.12(9)(10) do not hold, as it is shown in the following examples:

Example 3.15

Let $X=\{a,b,c\}$, and $\tau=P(X)$. We define an s-operation $\lambda:SO(X)\rightarrow P(X)$ as:

$$\lambda(A)=\begin{cases} A & \text{if } \ A=\{a,b\} \ or \ \{a,c\} \ or \ \phi \\ X & \text{Otherwise} \end{cases}.$$

$SO(X)=P(X)=\beta O(X)=\beta C(X)$.

$SO_{\lambda}(X)=\{\phi,\{a,b\},\{a,c\},X\}$.

$SO_{\lambda_{\beta c}}(X)=\{\phi,\{a,b\},\{a,c\},X\}$.

$SC_{\lambda_{\beta c}}(X)=\{\phi,\{b\},\{c\},X\}$. Now, if we let $A=\{b\}$ and $B=\{c\}$ then $\lambda_{\beta c}Cl(A)$ $=\{b\}$ and $\lambda_{\beta c}Cl(B)=\{c\}$, but $\lambda_{\beta c}Cl(A\cup B)=X$, where $A\cup B=\{b,c\}$. Hence we get $\lambda_{\beta c}Cl(A\cup B)\neq \lambda_{\beta c}Cl(A)\cup\lambda_{\beta c}Cl(B)$.

Example 3.16

Let $X=\{a,b,c\}$, and $\tau=\{\phi,\{a\},\{b\}\{a,b\},X\}$, We define an s-operation $\lambda:SO(X)\rightarrow P(X)$ as:

$$\lambda(A)=\begin{cases} A & \text{if } \ b\in A \\ X & \text{Otherwise} \end{cases}.$$

$SO(X)=\{\phi,\{a\},\{b\},\{a,b\},\{a,c\},\{b,c\},X\}=\beta O(X)$.

$SC(X)=\{\phi,\{a\},\{b\},\{c\},\{a,c\},\{b,c\},X\}=\beta C(X)$.

$SO_{\lambda}(X)=\{\phi,\{b\},\{a,b\},\{b,c\},X\}$

$SO_{\lambda_{\beta c}}(X)=\{\phi,\{b\},\{a,b\},\{b,c\},X\}$

$SC_{\lambda_{\beta c}}(X)=\{\phi,\{a\},\{c\}\{a,c\},X\}$. Now, if we let $A=\{a\}$ and $B=\{b\}$ then $\lambda_{\beta c}Cl(A)=\{a\}$ and $\lambda_{\beta c}Cl(B)=X$, but $\lambda_{\beta c}Cl(A\cap B)=\phi$, where $A\cap B=\phi$. Hence we get $\lambda_{\beta c}Cl(A\cap B)\neq \lambda_{\beta c}Cl(A)\cap\lambda_{\beta c}Cl(B)$.

Proposition 3.17

For a subset A of a topological space (X,τ), $\lambda^{*}Cl(A)\subseteq\lambda_{\beta c}Cl(A)$.

Proof. Let $x\in\lambda^{*}Cl(A)$ then $A\cap V\neq\phi$, for every $V\in SO_{\lambda}(X)$ such that $x\in V$ by Proposition 1.2. Then $A\cap V\neq\phi$, for every $V\in SO_{\lambda_{\beta c}}(X)$. Then $x\in\lambda_{\beta c}Cl(A)$ by Proposition 3.13. Hence $\lambda^{*}Cl(A)\subseteq\lambda_{\beta c}Cl(A)$. Finally, we have $\lambda^{*}Cl(A)\subseteq\lambda_{\beta c}Cl(A)$.

The converse of the Proposition 3.17 is not true in general as shown by following example.

Remark 3.18

In Example 3.3 if we let $A=\{a\}$, then $\lambda^{*}Cl(A)=\{a\}$, but $\lambda_{\beta c}Cl(A)=X$, but $\lambda_{\beta c}Cl(A)\nsubseteq\lambda^{*}Cl(A)$.

Definition 3.19

Let A be a subset of a topological space (X,τ). A point $x\in X$ is said to be $\lambda_{\beta c}$-interior point of A, if there exists a $\lambda_{\beta c}$-open set U containing x such that $U\subseteq A$. The set of all $\lambda_{\beta c}$-interior points of A is called $\lambda_{\beta c}$-interior of A and is denoted by $\lambda_{\beta c}Int(A)$.

Proposition 3.20

For subsets A, B of a topological space (X, τ), the following statements hold.

(1) $\lambda_{\beta c}Int(A)$ is the union of all $\lambda_{\beta c}$-open sets which are contained in A.

(2) $\lambda_{\beta c}Int(A)$ is a $\lambda_{\beta c}$-open set in X.

(3) $\lambda_{\beta c}Int(A) \subseteq A$.

(4) $\lambda_{\beta c}Int(A)$ is the largest $\lambda_{\beta c}$-open set contained in A.

(5) A is $\lambda_{\beta c}$-open set if and only if $\lambda_{\beta c}Int(A) = A$.

(6) $\lambda_{\beta c}Int(\lambda_{\beta c}Int(A)) = \lambda_{\beta c}Int(A)$.

(7) If $A \subseteq B$, then $\lambda_{\beta c}Int(A) \subseteq \lambda_{\beta c}Int(B)$.

(8) $\lambda_{\beta c}Int(\phi) = \phi$ and $\lambda_{\beta c}Int(X) = X$.

(9) If $A \cap B = \phi$, then $\lambda_{\beta c}Int(A) \cap \lambda_{\beta c}Int(B) = \phi$.

(10) $\lambda_{\beta c}Int(A) \cup \lambda_{\beta c}Int(B) \subseteq \lambda_{\beta c}Int(A \cup B)$.

(11) $\lambda_{\beta c}Int(A \cap B) \subseteq \lambda_{\beta c}Int(A) \cap \lambda_{\beta c}Int(B)$.

Proof. Obvious.

In general the equalities of (9), (10) and (11) of the above proposition does not hold, as it is shown in the following examples:

Example 3.21

Let $X = \{a,b,c\}$, and $\tau = \{\phi,\{a\},\{b\},\{a,b\},X\}$, $\tau^c = \{\phi,\{c\},\{a,c\},\{b,c\},X\}$. We define an s-operation $\lambda : SO(X) \to P(X)$ as:

$$\lambda(A) = \begin{cases} A & \text{if } A = \{a,b\} \\ Cl(A) & \text{Otherwise} \end{cases}.$$

$SO(X)=\{\phi,\{a\},\{b\},\{a,b\},\{a,c\},\{b,c\},X\}=\beta O(X).$

$SC(X)=\{\phi,\{a\},\{b\},\{c\},\{a,c\},\{b,c\},X\}=\beta C(X).$

$SO_{\lambda}(X)=\{\phi,\{a,b\},X\}.$

$SO_{\lambda_{\beta c}}(X)=\{\phi,\{a,b\},X\}.$

If $A=\{a\}$ and $B=\{b\}$, then $\lambda_{\beta c}Int(A)=\phi=\lambda_{\beta c}Int(B)$ but $\lambda_{\beta c}Int(A\cup B)=\{a,b\}$, where $A\cup B=\{a,b\}$. Thus $\lambda_{\beta c}Int(A\cup B)\neq\lambda_{\beta c}Int(A)\cup\lambda_{\beta c}Int(B)$.

Example 3.22

Let $X=\{a,b,c\}$, and $\tau=\{\phi,\{a\},\{b\},\{a,b\},X\}$, $\tau^{c}=\{\phi,\{c\},\{a,c\},\{b,c\},X\}$.

We define an s-operation $\lambda:SO(X)\to P(X)$ as:

$$\lambda(A)=\begin{cases}A & \text{if } A\in SO(X)\\ X & \text{Otherwise}\end{cases}.$$

$SO(X)=\{\phi,\{a\},\{b\},\{a,b\},\{a,c\},\{b,c\},X\}=\beta O(X).$

$SC(X)=\{\phi,\{a\},\{b\},\{c\},\{a,c\},\{b,c\},X\}=\beta C(X).$

$SO_{\lambda}(X)=\{\phi,\{a\},\{b\},\{a,b\},\{a,c\},\{b,c\},X\}.$

$SO_{\lambda_{\beta c}}(X)=\{\phi,\{a\},\{b\},\{a,b\},\{a,c\},\{b,c\},X\}.$

Let $A=\{a,c\}$ and $B=\{b,c\}$. Then $\lambda_{\beta c}Int(A)=\{a,c\}$ and $\lambda_{\beta c}Int(B)=\{b,c\}$, but $\lambda_{\beta c}Int(A\cap B)=\lambda_{\beta c}Int(\{c\})=\phi\neq\{a,c\}\cap\{b,c\}=\{c\}$ where $A\cap B=\{c\}$. Therefore $\lambda_{\beta c}Int(A\cap B)\neq\lambda_{\beta c}Int(A)\cap\lambda_{\beta c}Int(A)$.

Remark 3.23

1) In Example 3.3 if we let $A=\{a,b\}$ and $B=\{b,c\}$ then $\lambda_{\beta c}Int(A)\cap\lambda_{\beta c}Int(B)=\phi$, but $A\cap B=\{b\}\neq\phi$.

2) In Example 3.3 if we let $A=\{b,c\}$ and $B=\{a\}$, then $\lambda_{\beta c}Int(A)=\phi=\lambda_{\beta c}Int(B)$. But $A\neq B$.

Proposition 3.24

For a subset A of a topological space (X,τ), $\lambda_{\beta c}Int(A)=A\setminus\lambda_{\beta c}d(X\setminus A)$.

Proof. If $x\in A\setminus\lambda_{\beta c}d(X\setminus A)$ then $x\in A$ and $x\notin\lambda_{\beta c}D(X\setminus A)$ and so there exists a $\lambda_{\beta c}$-open set U containing x such that $U\cap X\setminus A=\phi$. Then $x\in U\subseteq A$, hence $x\in\lambda_{\beta c}Int(A)$, this implies that $A\setminus\lambda_{\beta c}D(X\setminus A)\subseteq\lambda_{\beta c}Int(A)$. On the other hand, if $x\in\lambda_{\beta c}Int(A)$ then $x\notin\lambda_{\beta c}D(X\setminus A)$ since $\lambda_{\beta c}Int(A)$ is $\lambda_{\beta c}$-open and $\lambda_{\beta c}Int(A)\cap X\setminus A=\phi$. Hence $\lambda_{\beta c}Int(A)=A\setminus\lambda_{\beta c}D(X\setminus A)$.

Proposition 3.25

For any subset A of a topological space (X,τ), The following statements are true.

(1) $X\setminus\lambda_{\beta c}Int(A)=\lambda_{\beta c}Cl(X\setminus A)$.

(2) $\lambda_{\beta c}Cl(A)=X\setminus\lambda_{\beta c}Int(X\setminus A)$.

(3) $X\setminus\lambda_{\beta c}Cl(A)=\lambda_{\beta c}Int(X\setminus A)$.

(4) $\lambda_{\beta c}Int(A)=X\setminus\lambda_{\beta c}Cl(X\setminus A)$.

Proof. (1) $X\setminus\lambda_{\beta c}Int(A)=X\setminus(A\setminus\lambda_{\beta c}D(X\setminus A))=(X\setminus A)$ ب $\lambda_{\beta c}D(X\setminus A)$ $=\lambda_{\beta c}Cl(X\setminus A)$.

(2) We have $X\setminus\lambda_{\beta c}Int(A)=\lambda_{\beta c}Cl(X\setminus A)$ by(1), replace A by $X\setminus A$ then

$X\setminus\lambda_{\beta c}Int(X\setminus A)=\lambda_{\beta c}Cl(A)$.

(3) We have $X \setminus \lambda_{\beta c}Int(X \setminus A) = \lambda_{\beta c}Cl(A)$ by(2), complement both sides then

$X \setminus \lambda_{\beta c}Cl(A) = \lambda_{\beta c}Int(X \setminus A)$.

(4) We have $X \setminus \lambda_{\beta c}Cl(A) = \lambda_{\beta c}Int(X \setminus A)$ by(3), replace A by $X \setminus A$ then

$\lambda_{\beta c}Int(A) = X \setminus \lambda_{\beta c}Cl(X \setminus A)$.

Proposition 3.26

For a subset A of a topological space (X, τ), $\lambda_{\beta c}Int(A) \subseteq \lambda^{*}Int(A)$.

Proof.Obvious.

The converse of Proposition 3.26, is not true in general, we can show by the following example:

Example 3.27

In Example 3.3 if we let $A=\{a,b\}$ then $\lambda_{\beta c}Int(A)=\phi$ and $\lambda^{*}Int(A)=\{a,b\}$. Therefore $\lambda^{*}Int(A) \not\subseteq \lambda_{\beta c}Int(A)$.

Corollary 3.28

If A is a subset of a topological space (X,τ), then $\lambda_{\beta c}Int(A) \subseteq \lambda^{*}Int(A) \subseteq A \subseteq \lambda^{*}Cl(A) \subseteq \lambda_{\beta c}Cl(A)$.

Proof. Obvious.

Theorem 3.29

Let A, B be subsets of X. If $\lambda : SO(X) \rightarrow P(X)$ is a λ-regular s-operation Then:

(1) $\lambda_{sc}d(A \cup B) = \lambda_{sc}d(A) \cup \lambda_{sc}d(B)$.

(2) $\lambda_{sc}Cl(A \cup B) = \lambda_{sc}Cl(A) \cup \lambda_{sc}Cl(B)$.

(3) $\lambda_{sc}Int(A \cap B) = \lambda_{sc}Int(A) \cap \lambda_{sc}Int(B)$.

Proof. (1) $\lambda_{sc}D(A)\cup\lambda_{sc}D(B)\subseteq\lambda_{sc}D(A\cup B)$ by Proposition 3.4. Let $x\in$ $\lambda_{sc}D(A\cup B)$, if $x\notin\lambda_{sc}D(A)\cup\lambda_{sc}D(B)$ then $x\notin\lambda_{sc}D(A)$ and $x\notin\lambda_{sc}D(B)$ then there exist $\lambda_{\beta c}$-open sets U and V contain x such that $(A\setminus\{x\})\cap U\neq\phi$ and $(B\setminus\{x\})\cap V\neq\phi$, then $((A\cup B)\setminus\{x\})\cap(U\cap V)=\phi$, but λ is λ-regular then $U\cap V$ is $\lambda_{\beta c}$-open by Proposition 2.11, so $x\notin\lambda_{\beta c}D(A\cup B)$ a contradiction. Thus $\lambda_{\beta c}D(A)\cup\lambda_{\beta c}D(B)\subseteq\lambda_{\beta c}D(A\cup B)$. Hence we get

$\lambda_{\beta c}D(A\cup B)=\lambda_{\beta c}D(A)\cup\lambda_{\beta c}D(B)$.

(2) $\lambda_{\beta c}Cl(A\cup B)=(A\cup B)\cup\lambda_{\beta c}D(A\cup B)$ by Proposition 3.12(7). But

$\lambda_{\beta c}D(A\cup B)=\lambda_{\beta c}D(A)\cup\lambda_{\beta c}D(B)$ by (1), therefore $\lambda_{\beta c}Cl(A\cup B)=$ $(A\cup B)\cup(\lambda_{\beta c}D(A)\cup\lambda_{\beta c}D(B))$, this implies that $\lambda_{\beta c}Cl(A\cup B)=$ $((A\cup\lambda_{\beta c}D(A))\cup(B\cup\lambda_{\beta c}D(B))=\lambda_{\beta c}Cl(A)\cup\lambda_{\beta c}Cl(B)$ by Proposition 3.12(7).

(3) $\lambda_{\beta c}Int(A\cap B)=(A\cap B)\setminus\lambda_{\beta c}D(X\setminus(A\cap B))$ by Proposition 3.24. So

$\lambda_{\beta c}Int(A\cap B)=(A\cap B)\setminus\lambda_{\beta c}D(X\setminus A\cup X\setminus B)$, but $\lambda_{\beta c}D(X\setminus A\cup X\setminus B)$ $=\lambda_{\beta c}D(X\setminus A)\cup\lambda_{\beta c}D(X\setminus B)$ by(1), this implies $\lambda_{\beta c}Int(A\cap B)=$ $(A\cap B)\setminus(\lambda_{\beta c}D(X\setminus A)\cup\lambda_{\beta c}D(X\setminus B))$. Therefore $\lambda_{\beta c}Int(A\cap B)=$ $A\setminus\lambda_{\beta c}D(X\setminus A)\cup B\setminus\lambda_{\beta c}D(X\setminus B)=\lambda_{\beta c}Int(A)\cap\lambda_{\beta c}Int(B)$ by Proposition 3.24.

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پوختە

لەم توێژینەوەیەدا چەمکی کۆمەڵەیەکی کراوە لە جۆری($\lambda_{\beta c}$)دەناسێنین لە بۆشاییە تۆپۆلۆجیەکاندا وە لێکۆڵینەوەی تایبە تمەندیە تۆپۆلۆجیەکان دەکەین وەك کۆمەڵەی سنوردار لە جۆری($\lambda_{\beta c}$)و کۆمەڵەی داخراو لە جۆری ($\lambda_{\beta c}$) وە کۆمەڵەی ناوەکی لە جۆری ($\lambda_{\beta c}$) بەبەکارهێنانی کۆمەڵەی کراوە لەجۆری ($\lambda_{\beta c}$) .

الملخّص

في هذه البحث نقدم مفهوم المجموعات المفتوحة من نوع($\lambda_{\beta c}$) في الفضاءات التبولوجية و ندرس الصفيات التبولوجيةلهذه المجموعات مثل مجموعة الاشتقاق من نوع ($\lambda_{\beta c}$)، مجموعة الإنغلاق من نوع ($\lambda_{\beta c}$)و المجموعة الداخلية من نوع ($\lambda_{\beta c}$)باستخدام المجموعةالمفتوحة من النوع ($\lambda_{\beta c}$) .