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Journal of University of Garmian

https://doi.org/10.24271/garmian.196363

Using Interactive Techniques and New Geometric Average Techniques to Solve MOLFPP

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Article Info

Received: August, 2019

Revised:August,2019

Accepted:September,2019

Keywords

MOLFPP, Interactive Techniques, New Geometric Average Techniques, Short-Hierarchical Technique

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Abstract

In this paper, we used an Interactive Techniques and New Geometric Average Techniques to solve multi-objective linear fractional programming problems (MOLFPP), and algorithms suggested for them, then solving this by Short-Hierarchical Technique. The computer application for algorithms is tested on a number of numerical examples. The results which are obtained by above techniques compared, and indicate that the results obtained by Interactive Techniques are better than others.

1.1 Introduction

Linear fraction maximum problems (i.e. ratio objective that have numerator and denominator) have attracted considerable research and interest, since they are useful in production planning, financial and corporative planning, health care and hospital planning. Several methods to solve such problems are proposed in (1962)[3]. Their method depends on transforming the linear fractional programming to an equivalent linear program. Sing (1981) in his paper did a useful study about the optimality condition in fractional programming [6]. Sadiq in (2005) studied the MOLPP by Short-Hierarchical [4]. Also Sulaiman and Nawkhass in (2015) studied the MOLFPP by Short-Hierarchical [10]. Also In (1993) Abdil-kadir and Sulaiman[1] studied the MOFPP. Salih in (2010) studied the MOLFPP [7]. Sulaiman and Abdulrahim in (2013) using transformation technique to solve MOLFPP [8]. Also Sulaiman and Abdulrahim in (2014) new arithmetic average technique to solve MOLFPP and it is comparison with other techniques[9]. Abdulrahim and Abdulah in (2015) Interactive techniques to solve MOLPP [2]. Sulaiman, Abdullah and Abdulla in (2015) Optimal geometric average to solve MOQPP [11].

In order to extend this work we have defined a MOLFPP and investigated the algorithm to solve fractional programming problem for multi-objective function, irrespective of the number of objectives with less computational burden and suggest an Interactive Techniques and New geometric average Techniques of objective functions, to generate the best optimal solution and also solved the problems by Short-Hierarchical. The computer application of our algorithm has also been discussed by solving a numerical example. Finally we have shown results and comparisons between different techniques.

Linear Fractional Programming Problem

The mathematical programming problem for LFPP can be formulated as follows:

$$\text{Maximize (Minimize) } Z = \frac{(c^t x + \gamma)}{(d^t x + \beta)}$$

Subject to:

$$x \in X = \left\{ x;\ \begin{matrix} AX \leq b \\ AX \geq b \\ AX = b \end{matrix} \right\}$$

Where $x \in R^n$, A is an $m \times n$ matrix; c and d are $n-$vectors; $b \in R^m$ and γ, β are scalar constants. Moreover $d^t x + \beta > 0$ everywhere in X [12].

1.1

1.2 Multi-Objective Linear Fractional Programming Problem

Multi-Objective function that are the ratio of two linear objective functions are said to be MOLFPP ([1],[8],[9],[10]) then can be defined:

$$\left.\begin{aligned} Max. Z_1 &= \frac{c_1^t x + \gamma_1}{d_1^t x + \beta_1} \\ Max. Z_2 &= \frac{c_2^t x + \gamma_2}{d_2^t x + \beta_2} \\ &\ \ \vdots \\ Max. Z_r &= \frac{c_r^t x + \beta_r}{d_r^t x + \beta_r} \\ Min. Z_{r+1} &= \frac{c_{r+1}^t x + \beta_{r+1}}{d_{r+1}^t x + \beta_{r+1}} \\ &\ \ \vdots \\ Min. Z_s &= \frac{c_s^t x + \gamma_s}{d_s^t x + \beta_s} \end{aligned}\right] \quad (3.1)$$

Subject to:

$$Ax = b \qquad (3.2)$$

$$x \geq 0 \qquad (3.3)$$

Where r is the number of objective function that to be maximized, s is the number of objective functions that is to be maximized and minimized and $s - r$ is the number of objerctive function that is to be minimized, other symbols have the same meaning as previously mentioned, for more details see[10], Where b is $m-$dimensional vector of constants, x is $n-$dimensional vector of decision variables and A is a $m \times n$ matrix of constants other symbols have the same meaning as before ([8],[9],[10]).

1.3 Solving MOLFPP by Using New Geometric Average Techniques

We formulate the combined objective function. To solve MOLFPP by New Geometric Average Techniques (Geometric Arithmetic Average Technique and New Geometric Arithmetic Average Technique) respectively consider below:

$$\left.\begin{aligned} &Max.Z = \frac{SM - SN}{GV_2} \\ &GV_2 = \frac{N_1 + N_2}{2} \quad \text{is Geometric Arithmetic Average} \end{aligned}\right\} \quad (4.5)$$

$$\left.\begin{aligned} &\text{Or } Max.Z = \frac{SM - SN}{GV_s} \\ &GV_s = \frac{N_1 + N_2}{s} \quad \text{is New Geometric Arithmetic Average} \end{aligned}\right\} \quad (4.6)$$

$$SM = \sum_{i=1}^{r} Z_i, \forall\, i = 1,2,\ldots,r. \,\&\, SN = \sum_{i=r+1}^{s} Z_i, \forall\, i = r+1, r+2, \ldots, s.$$

δM_i: the maximum value of the i th objective function

δN_i : the minimum value of the i th objective function

$N_1 = \sqrt[r]{\min\{\alpha M_i\}}$ & αM_i = Absolute Value of $(\delta M_i) = |\delta M_i|, \forall\, i = 1,2,\ldots,r.$

$N_2 = \sqrt[s-r]{\min\{\alpha M_i\}}$ & αN_i = Absolute Value of $(\delta N_i) = |\delta N_i|, \forall\, i = r+1, r+2, \ldots, s.$

1.5 Algorithm :(New Geometric Average Techniques)

An algorithm for obtaining the optimal solution for the MOLFPP defined in equation (3.1) can be summarized as follows:

Step1: Assign arbitrary values to each of the individual objective functions which are to be maximized or minimized.

Step2: Solve the first objective function by the Modified Simplex Method ([5], [7]), for linear fractional programming subject to constraints.

Step3: Check the feasibility of the solution obtained in step2, if it is feasible then go to step4, otherwise use dual Simplex Method [5] to remove infeasibility.

Step4: Assign a name to the optimum value of the objective function Z_i, say δ_i for $i = 1,2,3,\ldots,s$ as before.

Step5: Select N_1, N_2 and calculated GV_2, GV_s using formula (4.4) or (4.5).

Step6: Optimize the combined objective function under the same constraints (3.2) and (3.3).

1. 6 Solving MOLFPP by Using Interactive Techniques

We formulate the combined objective function. To solve MOLFPP by Interactive techniques (Interactive Arithmetic Average Technique and New Interactive Arithmetic Average Technique) respectively consider below:

$$\left.\begin{aligned} &Max.Z = \frac{SM - SN}{AV_2} \\ &AV_2 = \frac{M_1 + M_2}{2} \quad \text{is Interactive Arithmetic Average} \end{aligned}\right\} \qquad (6.6)$$

$$\left.\begin{aligned} &\text{Or } Max.Z = \frac{SM - SN}{AV_s} \\ &AV_s = \frac{M_1 + M_2}{s} \quad \text{is New Interactive Arithmetic Average} \end{aligned}\right\} \qquad (6.7)$$

$$SM = \sum_{i=1}^{r} Z_i, \forall\, i = 1,2,\dots,r. \;\&\; SN = \sum_{i=r+1}^{s} Z_i, \forall\, i = r+1, r+2, \dots, s.$$

δM_i: the maximum value of the i th objective function

δN_i : the minimum value of the i th objective function

$$M_1 = \frac{\min\{\alpha M_i\}}{|D_1|\,(\max[\sin\theta_{12}, \cos\theta_{12}])} \;\&\; \alpha M_i = \text{Absolute Value of } (\delta M_i) = |\delta M_i|, \forall\, i = 1,2,\dots,r.$$

$$M_2 = \frac{\min\{\alpha N_i\}}{|D_2|\,(\max[\sin\theta_{12}, \cos\theta_{12}])} \;\&\; \alpha N_i = \text{Absolute Value of } (\delta N_i) = |\delta N_i|, \forall\, i = r+1, r+2, \dots, s.$$

$$D_1 = \min[(c_i^t x + \gamma_i)/(d_i^t x + \beta_i)], \forall\, i = 1,2,\dots,r.$$

$$D_2 = \min[(c_i^t x + \gamma_i)/(d_i^t x + \beta_i)], \forall\, i = r+1, r+2, \dots, s.$$

$|D_1| =$ Length of the $(\min[(c_i^t x + \gamma_i)/(d_i^t x + \beta_i)], \forall\, i = 1,2,\dots,r)$ after change to linear

$|D_2| =$ Length of the$(\min[(c_i^t x + \gamma_i)/(d_i^t x + \beta_i)], \forall\, i = r+1, r+2, \dots, s)$after change to linear

$$\cos\theta_{12} = \frac{D_1 D_2}{|D_1||D_2|}, \sin\theta_{12} = \sqrt{1 - (\cos\theta_{12})^2} \;\; ([2],[13]).$$

Subject to the some constraints (3.2) and (3.3).

1.7 Algorithm :(Interactive Techniques)

An algorithm for obtaining the optimal solution for the MOLFPP defined in equation (3.1) can be summarized as follows:

Step1, step2, step3and step4 are the same as given in algorithm in the section 1.5 as before. **Step5:** Select M_1, M_2 and calculated AV_2, AV_s using formula (6.6) or (6.7). **Step6:** Optimize the combined objective function under the same constraints (3.2) and (3.3).

1.8 Numerical Examples

In this section, we present numerical examples

Example 8.1: Solve the following MOLFPP

$Max.Z_1 = (5x_1 + 3x_2)/(x_1 + x_2 + 1)$
$Max.Z_2 = (9x_1 + 5x_2)/(3x_1 + 3x_2 + 3)$
$Max.Z_3 = (3x_1 - 4x_2)/(x_1 + x_2 + 1)$
$Max.Z_4 = (3x_1 + 2x_2)/(2x_1 + 2x_1 + 2)$
Subject to:

$$2x_1 + 4x_2 \geq 8$$
$$x_1 + x_2 \leq 3$$
$$x_1 + 2x_2 \leq 10$$
$$2x_1 + x_2 \leq 5$$
$$x_1 \leq 2$$
$$x_1, x_2 \geq 0$$

Solution 8.1: After finding the value of each of individual objective functions of example 8.1 the results obtained by Modified Simplex Method ([5], [7]) are given and the numerical results as below in table 1:

Table 1: Results of example 8.1 by using Modified Simplex Method

i	Z_i	x_i	δ_i	$\alpha M_i = \lvert\delta M_i\rvert$ $\forall\, i = 1,2,\dots,r$	$\alpha N_i = \lvert\delta N_i\rvert$ $\forall\, i = r+1, r+2,\dots,s$
1	13/4	(2,1)	13/4	13/4	
2	23/12	(2,1)	23/12	23/12	
3	1/2	(2,1)	1/2	1/2	
4	1	(2,1)	1	1	

The solution for example 8.1 by a Short-Hierarchical, it cannot solve and The solution for example 8.1 when applying algorithm in the section 1.5 by using Geometric Arithmetic Average Technique and New Geometric Arithmetic Average Technique respectively is the same optimal solution shown in table 1, then the combined objective linear fractional function is:

$$(*)\ SM = \sum_{i=1}^{r} Z_i = \sum_{i=1}^{4} Z_i = \frac{(75x_1 + 10x_2)}{(6x_1 + 6x_2 + 6)}, SN = \sum_{i=r+1}^{s} Z_i = 0$$

$$N_1 = \frac{1501}{1785}, N_2 = 0, GV_2 = \frac{N_1 + N_2}{2} = \frac{732}{1741}, GV_s = \frac{N_1 + N_2}{s} = \frac{366}{1741}$$

$$Max.Z = \frac{SM - SN}{GV_2} = \frac{(130575x_1 + 17410x_2)}{(4392x_1 + 4392x_2 + 4392)} \ \& \ Max.Z = \frac{SM - SN}{GV_s} = \frac{(130575x_1 + 17410x_2)}{(2196x_1 + 2196x_2 + 2196)}$$

After solving it subject to the same constraints as before respectively, we get

$Max.Z = 15.86$ and

$x_1 = 2,\ x_2 = 1\&\ Max.Z = 31.71$

and $x_1 = 2,\ x_2 = 1$

The solution for example 8.1 when applying algorithm in the section 1.7 by using Interactive Arithmetic Average Technique and New Interactive Arithmetic Average Technique respectively is the same optimal solution shown in table 1, then the combined objective linear fractional function is:

$$SM = \sum_{i=1}^{r} Z_i = \sum_{i=1}^{4} Z_i = \frac{(75x_1 + 10x_2)}{(6x_1 + 6x_2 + 6)}, SN = \sum_{i=r+1}^{s} Z_i = 0$$
$$M_1 = \frac{137}{2821}, M_2 = 0, AV_2 = \frac{M_1 + M_2}{2} = \frac{137}{5642}, \ AV_s = \frac{M_1 + M_2}{s} = \frac{137}{11284}$$
$$Max.Z = \frac{SM - SN}{AV_2} = \frac{(423150x_1 + 56420x_2)}{(822x_1 + 822x_2 + 822)} \ \& \ Max.Z = \frac{SM - SN}{AV_s} = \frac{(846300x_1 + 112840x_2)}{(822x_1 + 822x_2 + 822)}$$

After solving it subject to the same constraints as before respectively, we get

$Max.Z = 274.5499$ and $x_1 = 2,\ x_2 = 1$ & $Max.Z = 549.0998$ and $x_1 = 2,\ x_2 = 1$

Example 8.2: Solve the following MOLFPP

$$Max.Z_1 = (3x_1 - 2x_2)/(x_1 + x_2 + 1)$$
$$Max.Z_2 = (9x_1 + 3x_2)/(x_1 + x_2 + 1)$$
$$Max.Z_3 = (3x_1 - 5x_2)/(2x_1 + 2x_2 + 2)$$
$$Min.Z_4 = (-6x_1 + 2x_2)/(2x_1 + 2x_2 + 2)$$
$$Min.Z_5 = (-3x_1 - x_2)/(x_1 + x_2 + 1)$$

Subject to:

$$x_1 + x_2 \le 2$$
$$9x_1 + x_2 \le 9$$
$$x_1, x_2 \ge 0$$

Solution 8.2: After finding the value of each of individual objective functions of example 8.2 the results obtained by Modified Simplex Method ([5], [7]) are given and the numerical results as below in table 2:

Table 2: Results of example 8.2 by using Modified Simplex Method

i	Z_i	x_i	δ_i	$\alpha M_i = \lvert \delta M_i \rvert$ $\forall\, i = 1,2,\dots,r$	$\alpha N_i = \lvert \delta N_i \rvert$ $\forall\, i = r+1, r+2 \dots, s$
1	$3/2$	(1,0)	$3/2$	$3/2$	
2	$9/2$	(1,0)	$9/2$	$9/2$	
3	$3/4$	(1,0)	$3/4$	$3/4$	
4	$-3/2$	(1,0)	$-3/2$		$3/2$
5	$-3/2$	(1,0)	$-3/2$		$3/2$

The solution for example 8.2 by a Short-Hierarchical, $Max\,.Z = 6.75$ and $x_1 = 1,\ x_2 = 0$ and the solution for example 8.2 when applying algorithm in the section 1.5 by using Geometric Arithmetic Average Technique and New Geometric Arithmetic Average Technique respectively is the same optimal solution shown in table 2, then the combined objective linear fractional function is:

$$SM = \sum_{i=1}^{r} Z_i = \sum_{i=1}^{3} Z_i = \frac{(27x_1 - 3x_2)}{(2x_1 + 2x_2 + 2)}, SN = \sum_{i=r+1}^{s} Z_i = \sum_{i=4}^{5} Z_i = \frac{(-6x_1)}{(x_1 + x_2 + 1)}$$
$$N_1 = \frac{467}{514}, \quad N_2 = \frac{1079}{881}, GV_2 = \frac{N_1 + N_2}{2} = \frac{5009}{4696}, GV_s = \frac{N_1 + N_2}{s} = \frac{989}{2318}$$
$$Max.Z = \frac{SM - SN}{GV_2} = \frac{(183144x_1 - 14088x_2)}{(10018x_1 + 10018x_2 + 10018)} \ \& \ Max.Z = \frac{SM - SN}{GV_s} = \frac{(90402x_1 - 6954x_2)}{(1978x_1 + 1978x_2 + 1978)}$$

After solving it subject to the same constraints as before respectively, we get

$Max\ .Z = 9.14$ and $x_1 = 1,\ x_2 = 0$

& $Max\ .Z = 22.85$ and $x_1 = 1,\ x_2 = 0$

The solution for example 8.2 when applying algorithm in the section 1.7 by using Interactive Arithmetic Average Technique and New Interactive Arithmetic Average Technique respectively is the same optimal solution shown in table 2, then the combined objective linear fractional function is:

$$SM = \sum_{i=1}^{r} Z_i = \sum_{i=1}^{3} Z_i = \frac{(27x_1 - 3x_2)}{(2x_1 + 2x_2 + 2)}, \qquad SN = \sum_{i=r+1}^{s} Z_i = \sum_{i=4}^{5} Z_i = \frac{(-6x_1)}{(x_1 + x_2 + 1)}$$

$$M_1 = \frac{97}{3313}, \qquad M_2 = \frac{426}{2807}, AV_2 = \frac{M_1 + M_2}{2} = \frac{721}{7965}, AV_s = \frac{M_1 + M_2}{s} = \frac{123}{3397}$$

$$Max.Z = \frac{SM - SN}{AV_2} = \frac{(310635x_1 - 23895x_2)}{(1442x_1 + 1442x_2 + 1442)} \ \& \ Max.Z = \frac{SM - SN}{AV_s} = \frac{(132483x_1 - 10191x_2)}{(246x_1 + 246x_2 + 246)}$$

After solving it subject to the same constraints as before respectively, we get

$Max\ .Z = 107.7098$ and $x_1 = 1,\ x_2 = 0$

& $Max\ .Z = 269.2744$ and $x_1 = 1,\ x_2 = 0$

1.9 Comparison of the Numerical Results

Now, we are going to comparison the numerical results which are obtained of the examples as below in table 3:

Table 3: Comparison between results of the numerical techniques

Techniques		Example 8.1	Example 8.2:
Short-Hierarchical Technique		It not solve	$Max.Z = 6.75$ $x_1 = 1 \& x_2 = 0$
New Geometric Average Techniques	Geometric Arithmetic Average Technique	$Max.Z = 15.86$ $x_1 = 2 \& x_2 = 1$	$Max\ .Z = 9.14$ $x_1 = 1 \& x_2 = 0$
	New Geometric Arithmetic Average Technique	$Max.Z = 31.71$ $x_1 = 2 \& x_2 = 1$	$Max\ .Z = 22.85$ $x_1 = 1 \& x_2 = 0$
Interactive Techniques	Interactive Arithmetic Average Technique	$Max.Z = 274.5499$ $x_1 = 2 \& x_2 = 1$	$Max\ .Z = 107.7098$ $x_1 = 1 \& x_2 = 0$
	New Interactive Arithmetic Average Technique	$Max.Z = 549.0998$ $x_1 = 2 \& x_2 = 1$	$Max\ .Z = 269.2744$ $x_1 = 1 \& x_2 = 0$

In table 3; it is clear that the results obtained in examples 8.1, 8.2 when using Interactive Techniques are better than other results which are obtained by using short-hierarchical Technique and New Geometric Arithmetic Average Techniques.

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