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Using Pseudoaffinity To Translation QFPP To LFPP

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Abstract

In this paper, deal we with the problem of optimizing the ratio of two quadratic functions subject to a set linear constraints with the additional restriction that the optimal solution should also translation quadratic fractional programming problem (QFPP) to linear fractional programming problem (LFPP) by using pseudoaffinity after solving by modified simplex method. And consequently a convergent algorithm has been developed in the following discussion. Numerical examples have been provided to support the theory, by using Matlab 2016.

Keywords: Translation QFPP, by Pseudoaffinity to LFPP, Modified Simplex Method.

1.1 Introduction

The quadratic fractional programming problems (QFPP) are the topic of great importance in nonlinear programming. They are useful in many fields such as production planning, financial and corporative planning, health care and hospital planning. In various applications of nonlinear programming, one often encounters the problem in which the ratio of given two functions is to be maximized or minimized. Several methods to solve such problems are proposed in (1962) Charnes and Cooper ([6], [13]), Linear fraction problems (i.e. ratio of objectives that have numerator and denominator) have attracted considerable research and interest, their method depends on transforming this LFPP to an equivalent linear program, showed that by a simple transformation the original LFPP can be reduced to an LPP that can therefore be solved using a regular simplex method for LP. It was found that this approach is very useful for mathematicians because most theoretical results developed in LP could be relatively easily expanded to include LFPP [4]. Sing (1981) [11] did a useful study about the optimality condition in FP. In (2007) Tantawy studied a feasible direction method to solve LFPP [17]. Also in (2010) Salih studied and developed a feasible direction method to solve LFPP which is defined by Tantawy and we have suggested an approach to solve the same problem by using the modified simplex method [14]. Khurana and Arora (2011) studied an algorithm for solving a QFP when some of its constraints are homogeneous ([9], [10]). Moreover, in (2008) Fukushima and Hayashi have been addressed QFPP with quadratic constraints [8]. Abdulrahim, (2011) studies on solving QPP with extreme points [2]. In (2013) Abdulrahim, solving QFPP via feasible direction development and modified simplex method [3]. In (2013) Sulaiman and Nawkhass they have study a new modified simplex method to solve QFPP and compared it to a traditional simplex method by using pseudoaffinity of quadratic fractional functions [13]. Also in (2005) Biggs worked on Nonlinear Optimization with Financial Applications [5]. In (2013) Sulaiman and Nawkhass they have study a solving QFPP by using the Wolfe's method and a modified simplex approach [12]. To extend this work, we have been defined QFPP and investigated new technique to convert the quadratic fractional objective function to linear fractional objective function by using pseudoaffinity to generate the best compromising optimal solution. In addition, the special cases of the problem will be solved by Modified Simplex Method after convert the objective function to the pseudoaffinity function.

1.2 Definitions and Theorems

Definition 2.1: Linear Fractional Programming Problem

The mathematical programming problem for LFPP can be formulated as follows:

$$\text{Maximize (Minimize) } Z = \frac{(c^T x + \gamma)}{(d^T x + \beta)}$$

Subject to:

$$x \in X = \left\{ x;\ \begin{matrix} AX \le b \\ AX \ge b \\ AX = b \end{matrix} \right\}$$

Where $x \in R^n$, A is an $m \times n$ matrix; c and d are $n-$vectors; $b \in R^m$ and γ, β are scalar constants. Moreover $d^T x + \beta > 0$ everywhere in X [17].

Definition 2.2: Quadratic Programming Problem

If the optimization problem is of the form

$$\text{Maximize (Minimize) } Z = \alpha + C^T x + \frac{1}{2} x^T G x$$

Subject to :

$$Ax \begin{bmatrix} \le \\ \ge \\ = \end{bmatrix} b$$

$$x \ge 0$$

Where

$A = (a_{ij})_{m \times n}$, matrix of cofficients, $\forall i = 1,2,\ldots,m$, and $j = 1,2,\ldots,n$, $b = (b_1, b_2, \ldots, b_m)^T$, $x = (x_1, x_2, \ldots, x_n)^T$, $C^T = (c_1, c_2, \ldots, c_n)^T$

, and $G = (g_{ii})_{n \times n}$ is a positive definite or positive semi-definite symmetric square matrix, and T is transposed then the constraints are linear and the objective function is quadratic. Such an optimization problem is said to be a QPP [1].

Definition 2.3: Quadratic Fractional Programming Problem

The mathematical programming problem for QFPP can be formulated as follows

$$\text{Maximize (Minimize) } Z = \frac{\alpha_1 + C_1^T x + \frac{1}{2} x^T G_1 x}{\alpha_2 + C_2^T x + \frac{1}{2} x^T G_2 x}$$

Subject to :

$$Ax \begin{bmatrix} \le \\ \ge \\ = \end{bmatrix} b$$

$$x \ge 0$$

Where G_1, G_2 are $(n \times n)$ matrix of coefficients with G_1, G_2 are symmetric matrixes. All vectors are assumed to be column vectors unless transposed(T), where x is an n-dimensional vector of decision variables, C_1, C_2 is the n-dimensional vector of constants, b is m-dimensional vector of constants, α_1, α_2 are scalars.

In this work the problem that has objective function is tried to be solved has the following form:

$$\text{Maximize (Minimize)} Z = \frac{(c^T x + \gamma)(e^T x + \delta)}{(d^T x + \beta)(f^T x + \varepsilon)} = \frac{\frac{(c^T x + \gamma)(e^T x + \delta)}{(d^T x + \beta)}}{(f^T x + \varepsilon)} = \frac{\frac{(c^T x + \gamma)(e^T x + \delta)}{(f^T x + \varepsilon)}}{(d^T x + \beta)}$$

Subject to:

$$Ax \begin{bmatrix} \le \\ \ge \\ = \end{bmatrix} b$$

$$x \ge 0$$

Where $x \in R^n$, A is an $m \times n$ matrix; c, e, d and f are n −vectors; $b \in R^m$ and $\gamma, \beta, \delta, \varepsilon$ are scalar constants. Moreover $f^T x + \varepsilon,\ d^T x + \beta > 0$ everywhere in x.

Definition 2.4: Pseudoaffinity of Quadratic Fractional Functions

In this section we are going to characterize the pseudoaffinity of quadratic fractional functions of the following kind:

$$\left. \begin{array}{l} f(x) = \dfrac{\frac{\alpha_1 + C_1^T x + \frac{1}{2} x^T G_1 x}{(d^T x + \beta)}}{(f^T x + \varepsilon)} = \dfrac{g(x)}{(f^T x + \varepsilon)} = \dfrac{\frac{(c^T x + \gamma)(e^T x + \delta)}{(d^T x + \beta)}}{(f^T x + \varepsilon)} \\ \text{Or } f(x) = \dfrac{\frac{\alpha_1 + C_1^T x + \frac{1}{2} x^T G_1 x}{(f^T x + \varepsilon)}}{(d^T x + \beta)} = \dfrac{g(x)}{(d^T x + \beta)} = \dfrac{\frac{(c^T x + \gamma)(e^T x + \delta)}{(f^T x + \varepsilon)}}{(d^T x + \beta)} \end{array} \right\} \quad (1)$$

Defined on the set $X = \{x \in \Re^n : (d^T x + \beta) > 0\}$, or $X = \{x \in \Re^n : (f^T x + \varepsilon) > 0\}$ where $G_1 \neq 0$ is a $n \times n$ symmetric matrix, $d, f, x, C_1 \in \Re^n$, $f, d \neq 0$, and $\alpha_1, \beta, \varepsilon, \delta, \gamma \in \Re$. Note that being G_1 symmetric, it is $G_1 \neq 0$ if and only if $v_0(G_1) \le n - 1$ [7].

Corollary 2.1: Consider function $f(x)$ defined in (1) and suppose that there exist $a_0, b_0, p_0 \in \Re$, $a_0 \neq 0$, such that $f(x)$ can be written in the following form:

$$f(x) = \frac{a_0 d^T x + b_0 + \frac{a_0 p_0}{(d^T x + \beta)}}{(f^T x + \varepsilon)}$$

$$\text{or } f(x) = \frac{a_0 f^T x + b_0 + \frac{a_0 p_0}{(f^T x + \varepsilon)}}{(d^T x + \beta)}$$

i) If $p_0 \leq 0$ then $g(x)$ is pseudoaffine on X.

ii) If $p_0 > 0$ then $g(x)$ is pseudoaffine on

$X_1 = \{x \in \Re^n : (d^T x + \beta) > \sqrt{p_0}\}$ and $X_2 = \{x \in \Re^n : 0 < (d^T x + \beta) < \sqrt{p_0}\}$[7].

In our studies we take special cases where $p_0 = 0$, then the function $g(x) = a_0 d^T x + b_0$ is pseudoaffine on X, $f(x) = \frac{a_0 d^T x + b_0}{(f^T x + \varepsilon)}$ but where $p_0 > 0$, then the function $g(x) = a_0 d^T x + b_0 + \frac{a_0 p_0}{(d^T x + \beta)}$ is not pseudoaffine on X, $f(x) = \frac{a_0 d^T x + b_0 + \frac{a_0 p_0}{(d^T x + \beta)}}{(f^T x + \varepsilon)}$ and linear fractional functions by adding constraints respectively then it can solve it by Modified Simplex Method, which is shown in numerical examples and result section 1.5.

1.3 Modified Simplex Method Development

Simplex method is developed by Dantzig in (1947). The Simplex method provides a systematic algorithm which consists of moving from one basic feasible solution (one vertex) to another in prescribed manner such that the value of the objective function is improved. This procedure of jumping from vertex to vertex is repeated. If the objective function is improved at each jump, then no basis can ever be repeated and there is no need to go to back to vertex already covered. Since the number of vertices is finite, the process must lead to the optimal vertex in a finite number of steps. The Simplex algorithm is an iterative (step by step) procedure for solving linear programming problems. It consists of:

I. Having a trail basic feasible solution to constraint equations.
II. Testing whether is an optimal solution.
III. Improving the first trial solution by a set of rules, and repeating the process till an optimal solution is obtained. For more details [15].

Modified Simplex method to solve linear fractional programming problem and to solve quadratic objective function can be written as the produced two linear functions (QPP) [16]. Using modified Simplex method to solve the numerical example to apply simplex process [16]. First we find Δ_{j1}, and Δ_{j2} from the coefficient of numerator and denominator of objective function respectively, by using the following formula:

$\Delta_{ji} = C_{ji} - C_{Bi} x_{ji}, i = 1,2, j = 1,2, \dots, m+n,$

$Z_1 = C_{B1} V_B + \gamma,\ Z_2 = C_{B2} V_B + \beta\ ,\gamma, \beta$ are constants, $Z = \frac{Z_1}{Z_2}$

In this approach we define the formula to find Δ_j from Z_1, Z_2, Δ_{j1} and Δ_{j2} as follows: $\Delta_j = Z_2 \Delta_{j1} - Z_1 \Delta_{j2}$. Here C_{ji} are the coefficients of the basic and non-basic variables in the objective function and , C_{Bi} are the coefficients of the basic variables in the objective function, $j = 1,2, \dots, m+n,\ i = 1,2$. For testing optimality solution must be all $\Delta_j \leq 0$ but here all Δ_j not lesser than zero, and then the solution is not optimal. Repeat the same approach to find next feasible solution.

1.4 Algorithm For Modified Simplex Method Of QFPP

An algorithm for solving QFPP by modified simplex method can be summarized as follows ([14], [16]):

Step1: Write the standard from of the problem, by introducing slack, Surplus and artificial variables to constraints, and write starting simplex table, after convert the quadratic fractional objective function to linear fractional objective function by using pseudoaffinity.

Step2: Write the Δ_j row in the starting simplex table as: $\Delta_j = Z_2 \Delta_{j1} - Z_1 \Delta_{j2}$

Step3: Use simplex process to find the solution.

Step4: Check the feasibility of the solution in step3, if is feasible then go to step5, otherwise use dual simplex method to remove infeasibility.

Step5: Check the solution for optimality if all $\Delta_j \leq 0$, then the solution is optimal, otherwise go to step3.

1.5 Numerical Example and Results

Example 1: We consider the following QFPP as

$Max.Z = \frac{9x_1^2+24x_1x_2+16x_2^2+6x_1+8x_2+1}{3x_1^2+7x_1x_2+4x_2^2+7x_1+9x_2+2}$

Subject to:

$x_1 + x_2 \le 6$

$8x_1 + 4x_2 \le 32$

$x_1 \le 3$

$x_2 \le 5$

$x_1, x_2 \ge 0$

Solution 1: Solving the example 1 by Modified Simplex Method, after convert the objective function to pseudoaffinity function as follows:

$Max.Z = \frac{\frac{9x_1^2+24x_1x_2+16x_2^2+6x_1+8x_2+1}{(3x_1+4x_2+1)}}{(x_1+x_2+2)}$ Where $G_1 = \begin{bmatrix} 18 & 24 \\ 24 & 32 \end{bmatrix}, C_1^T = \begin{bmatrix} 6 \\ 8 \end{bmatrix}, d^T = \begin{bmatrix} 3 \\ 4 \end{bmatrix}, \alpha_1 = 1, \beta = 1$

Then we get $g(x) = 3x_1 + 4x_2 + 1$ is pseudoaffinity function by corollary part (i) where $p_0 = 0$

And we get $Max.Z = \frac{3x_1+4x_2+1}{x_1+x_2+2}$. After finding the values of the objective function by Modified Simplex Method with used the same constraints, After 2 steps we obtained the initial table as follows in table 1. After three iterations, we obtained the result in the following table 2:

Table 1: Initial table for example 1 by Modified Simplex Method

			C_{j1}	3	4	0	0	0	0	
			C_{j2}	1	1	0	0	0	0	
B.V.	C_{B1}	C_{B2}	V_B	x_1	x_2	x_3	x_4	x_5	x_6	Min ratio
x_3	0	0	6	1	1	1	0	0	0	$6/1 = 6$
x_4	0	0	32	8	4	0	1	0	0	$32/4 = 8$
x_5	0	0	3	1	0	0	0	1	0	–
x_6	0	0	5	0	1	0	0	0	1	$5/1 = 5$
$Z_1 = 1$			Δ_{j1}	3	4	0	0	0	0	
$Z_2 = 2$			Δ_{j2}	1	1	0	0	0	0	
$Z = \frac{Z_1}{Z_2} = \frac{1}{2}$			Δ_j	5	7	0	0	0	0	

Table 2: Final table for example 1 by Modified Simplex Method

			C_{j1}	3	4	0	0	0	0	
			C_{j2}	1	1	0	0	0	0	
B.V.	C_{B1}	C_{B2}	V_B	x_1	x_2	x_3	x_4	x_5	x_6	Min ratio
x_5	0	0	2	0	0	-1	0	1	1	
x_2	4	1	5	0	1	0	0	0	1	
x_1	3	1	1	1	0	1	0	0	-1	
x_4	0	0	4	0	0	-8	1	0	4	
$Z_1 = 24$			Δ_{j1}	0	0	-3	0	0	-1	
$Z_2 = 8$			Δ_{j2}	0	0	-1	0	0	0	
$Z = \frac{Z_1}{Z_2} = \frac{24}{8} = 3$			Δ_j	0	0	0	0	0	-8	

After solving it by Modified Simplex Method, we get $Max.Z = 3$ and $x_1 = 1,\ x_2 = 5$

Example 2: We consider the following QFPP as

$$Max.Z = \frac{-8x_1^2+24x_2^2+32x_3^2+16x_1x_2+64x_2x_3+16x_2+16x_3+2}{16x_1^2+16x_2^2+32x_3^2+32x_1x_2+48x_2x_3+48x_1x_3+24x_1+24x_2+40x_3+8}$$

Subject to:

$$x_1 + 3x_2 + 2x_3 \le 9$$
$$3x_1 + 2x_2 + x_3 \le 8$$
$$2x_1 + x_2 + 3x_3 \le 7$$
$$x_1, x_2, x_3 \ge 0$$

Solution 2: Solving the example (2) by Modified Simplex Method, after convert the objective function to pseudoaffinity function as follows:

$$Max.Z = \frac{\frac{-8x_1^2+24x_2^2+32x_3^2+16x_1x_2+64x_2x_3+16x_2+16x_3+2}{(8x_1+8x_2+16x_3+4)}}{(2x_1+2x_2+2x_3+2)}$$

Where $G_1 = \begin{bmatrix} -16 & 16 & 0 \\ 16 & 48 & 64 \\ 0 & 64 & 64 \end{bmatrix}, C_1^T = \begin{bmatrix} 0 \\ 16 \\ 16 \end{bmatrix}, d^T = \begin{bmatrix} 8 \\ 8 \\ 16 \end{bmatrix}, \alpha_1 = 2, \beta = 4$

Then we get $g(x) = -x_1 + 3x_2 + 2x_3 + \frac{1}{2}$ is pseudoaffinity function by corollary part (i) where $p_0 = 0$. And we get $Max.Z = \frac{-x_1+3x_2+2x_3+\frac{1}{2}}{2x_1+2x_2+2x_3+2}$. After finding the values of the objective function by Modified Simplex Method with used the same constraints, After 2 steps we obtained the initial table as follows in table 1. After two iterations, we obtained the result in the following table 2:

Table 1: Initial table for example 2 by Modified Simplex Method

B.V.	C_{B1}	C_{B2}	V_B	x_1	x_2	x_3	x_4	x_5	x_6	Min ratio
			C_{j1}	-1	3	2	0	0	0	
			C_{j2}	2	2	2	0	0	0	
x_4	0	0	9	1	3	2	1	0	0	$9/3 = 3$
x_5	0	0	8	3	2	1	0	1	0	$8/2 = 4$
x_6	0	0	7	2	1	3	0	0	1	$7/1 = 7$
$Z_1 = \frac{1}{2}$			Δ_{j1}	-1	3	2	0	0	0	
$Z_2 = 2$			Δ_{j2}	2	2	2	0	0	0	
$Z = \frac{Z_1}{Z_2} = \frac{1}{4}$			Δ_j	-3	5	3	0	0	0	

Table 2: Final table for example 2 by Modified Simplex Method

B.V.	C_{B1}	C_{B2}	V_B	x_1	x_2	x_3	x_4	x_5	x_6	Min ratio
			C_{j1}	-1	3	2	0	0	0	
			C_{j2}	2	2	2	0	0	0	
x_2	3	2	3	$1/3$	1	$2/3$	$1/3$	0	0	
x_5	0	0	2	$7/3$	0	$-1/3$	$-2/3$	1	0	
x_6	0	0	4	$5/3$	0	$7/3$	$-1/3$	0	1	
$Z_1 = \frac{19}{2}$			Δ_{j1}	-2	0	0	-1	0	0	
$Z_2 = 8$			Δ_{j2}	$4/3$	0	$2/3$	$-2/3$	0	0	
$Z = \frac{Z_1}{Z_2} = \frac{19}{16}$			Δ_j	$-86/3$	0	$-19/3$	$-5/3$	0	0	

After solving it by Modified Simplex Method, we get $Max.Z = \frac{19}{16}$ and $x_1 = 0$, $x_2 = 3, x_3 = 0$

Example 3: We consider the following QFPP as

$$\text{Max.}\, Z = \frac{4x_1^2 + 4x_2^2 + 8x_1x_2 + 9x_1 + 9x_2 + 3}{2x_1^2 + 2x_2^2 + 4x_1x_2 + 5x_1 + 5x_2 + 2}$$

Subject to:

$$4x_1 - 2x_2 \le 20$$
$$3x_1 + 5x_2 \le 25$$
$$x_1, x_2 \ge 0$$

Solution 3: Solving the example 3 by Modified Simplex Method, after convert the objective function to pseudoaffinity function as follows:

$$\text{Max. } Z = \frac{\frac{4x_1^2+4x_2^2+8x_1x_2+9x_1+9x_2+3}{(x_1+x_2+2)}}{(2x_1+2x_2+1)}$$ Where $G_1 = \begin{bmatrix} 8 & 8 \\ 8 & 8 \end{bmatrix}, C_1^T = \begin{bmatrix} 9 \\ 9 \end{bmatrix}, d^T = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \alpha_1 = 3, \beta = 2$

Then we get $g(x) = (4x_1 + 4x_2 + 1) + \frac{1}{(x_1+x_2+2)}$ is pseudoaffinity function by corollary part (ii) where $p_0 > 0$. And we get

$$Max.Z = \frac{(4x_1+4x_2+1)+\frac{1}{(x_1+x_2+2)}}{(2x_1+2x_2+1)} = \frac{(4x_1+4x_2+1)}{(2x_1+2x_2+1)} + \frac{\frac{1}{(x_1+x_2+2)}}{(2x_1+2x_2+1)}$$

Here we have the remainder second part $c = (0,0)$ $d = (1,1)$, $\alpha = 1, \beta = 2$

$$\left(c - \frac{\alpha}{\beta} d\right) x + \frac{\alpha}{\beta} = \left[(0,0) - \left(\frac{1}{2}\right)(1,1)\right]\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \left(\frac{1}{2}\right) = -\frac{1}{2}x_1 - \frac{1}{2}x_2 + \frac{1}{2}$$

$$Max.Z = \frac{(4x_1+4x_2+1)}{(2x_1+2x_2+1)} + \frac{\left(-\frac{1}{2}x_1 - \frac{1}{2}x_2 + \frac{1}{2}\right)}{(2x_1+2x_2+1)} = \frac{\frac{7}{2}x_1 + \frac{7}{2}x_2 + \frac{3}{2}}{(2x_1+2x_2+1)}$$

After finding the values of the objective function by Modified Simplex Method with used the same constraints After 2 steps we obtained the initial table as follows in table 1. After two iterations, we obtained the result in the following table 2:

Table 1: Initial table for example 3 by Modified Simplex Method

			C_{j1}	$7/2$	$7/2$	0	0	
			C_{j2}	2	2	0	0	
B.V.	C_{B1}	C_{B2}	V_B	x_1	x_2	x_3	x_4	Min ratio
x_3	0	0	20	4	-2	1	0	$20/4 = 5$
x_4	0	0	25	3	5	0	1	$25/3 = 8.33$
$Z_1 = \frac{3}{2}$			Δ_{j1}	$7/2$	$7/2$	0	0	
$Z_2 = 1$			Δ_{j2}	2	2	0	0	
$Z = \frac{Z_1}{Z_2} = \frac{3}{2}$			Δ_j	$1/2$	$1/2$	0	0	

Table 2: Final table for example 3 by Modified Simplex Method

			C_{j1}	7/2	7/2	0	0	
			C_{j2}	2	2	0	0	
B.V.	C_{B1}	C_{B2}	V_B	x_1	x_2	x_3	x_4	Min ratio
x_1	7/2	2	75/13	1	0	5/26	1/13	
x_2	7/2	2	20/13	0	1	$-3/26$	2/13	
$Z_1=\frac{352}{13}$			Δ_{j1}	0	0	$-7/26$	$-21/26$	
$Z_2=\frac{203}{13}$			Δ_{j2}	0	0	$-2/13$	$-6/13$	
$Z=\frac{Z_1}{Z_2}=\frac{352}{203}$			Δ_j	0	0	$-1/26$	$-3/26$	

After solving it by Modified Simplex Method, we get $Max.Z=\frac{352}{203}$ and $x_1=\frac{75}{13}$, $x_2=\frac{20}{13}$

1.6 Comparison of the Numerical Results

Now, we are going to comparison the numerical results which are obtained of the examples as below in table 3:

Table 3: Comparison between results of the Objective Functions

Examples	Before Correct The Objective Function (QFPP)	After Correct The Objective Function (LFPP)
Example1	$Max.Z=3$ and $x_1=1, x_2=5$	$Max.Z=3$ and $x_1=1, x_2=5$
Example2	$Max.Z=\frac{19}{16}$ and $x_1=0, x_2=3, x_3=0$	$Max.Z=\frac{19}{16}$ and $x_1=0, x_2=3, x_3=0$
Example3	$Max.Z=\frac{3705}{1907}$ and $x_1=\frac{75}{13}$, $x_2=\frac{20}{13}$	$Max.Z=\frac{352}{203}$ and $x_1=\frac{75}{13}$, $x_2=\frac{20}{13}$

In the above table 3, we compare the result. It is notice that value of objective function in example 1, example 2, and example 3 they have same results when it solved by Modified Simplex Method after convert the objective function by Pseudoaffinity function.

1.7 Discussion

In this paper solved QFPP by the Modified Simplex Method after convert objective function by Pseudoaffinity function to found the maximum value of QFPP. The optimal solution must be at one of the points of the polygon of the feasible, sometimes it may be need to use corollary 2.2 part (ii) for finding best optimal solution for the problem. The comparison of this method is based on the value of the objective function, the study found that $Max.Z$ resulted of that method are same, therefore we can solve of QFPP by this method under our method and algorithm. Consequently reliable results have been found.

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الخلاصة

في هذا البحث، نتعامل مع نسبة أمثلية لأثنين من الدوال من الدرجة الثانية للقيود المجموعة الخطية و ايضا تحويل مشكلة كسور البرمجة من الدرجة الثانية الى مشكلة كسور البرمجة الخطية بأستخدام Pseudoaffinity وحلها بطريقة المبسطة المتطورة. و بالتالي تم تطوير خوارزمية متقاربة في المناقشة التالية. و قد تم تزويد أمثلة عددية لهذه النظرية بأستخدام ماتلاب 2016.

پوختە

لەم توێژینەوەیەدا ، مامەڵە لە گەڵ باشترین دوو نەخشەی ڕێژەیی بە توانای دووجایی بۆ بەربەستی کۆمەڵەی هێڵی وە هەر وەها گۆرینی کێشەی کەرتی توانی دووجایی بۆ کێشەی کەرتی هێڵی بە کارهێنانی Pseudoaffinity وە شیکارکردنی بە ڕێگای سادەی پەرەسێندراو . وە گفتگۆکردن لە سەر گەشەکردنی نزێککراوەی خوارزمیە دەکەین. وە نموونە ژمارەیەکان بە دەستمان کەوتووە بە بە کارهێنانی پرۆگرامی ماتلابی 2016.