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Contra $(\lambda,\gamma)^*$-Continuous Functions

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Abstract

In this paper, some types of continuous functions via s-operations are introduced such as Contra $(\lambda,\gamma)^*$ -Continuous Functions and investigated. Several properties of these functions are constructed.

1. Introduction

In 1979, Kasahara S[1], introduced operation compact spaces. In 1991, Ogata H.[2], defined operations on topological spaces and associated topology. In 1992, Rehman F.U., and Ahmad B.[3], defined Operations on topological spaces. In 1999, Dontchev J., and Noiri T[4],defined Contra-semi continuous functions. They defined a function $f: X \to Y$ to be contra-continuous if the preimage of every open set of Y is semi-closed in X. In 2003, Ahmad B., and Hussain S.[5], defined γ -Convergence in Topological Spaces. In 2007, Hussain S.[6], defined Gamma-Operations in Topological Spaces. In 2012, S.F.Namiq and A.B.Khalaf defined $(\lambda,\gamma)^*$-continuous functions[7]. We consider λ as a function defined on $SO(X)$ into $P(X)$ and $\lambda: SO(X) \to P(X)$ is called an s-operation if $V \subseteq \lambda(V)$ for each non-empty semi open set V [7]. It is assumed that $\lambda(\phi) = \phi$ and $\lambda(X) = X$ for any s-operation λ [7]. Let $\lambda: SO(X) \to P(X)$ be an s-operation, then a subset A of X is called a λ^* -open set[8] which is equivalent to λ -open set[9] and λs - open set[10],if for each $x \in A$ there exists a semi open set U such that $x \in U$ and $\lambda(U) \subseteq A$. We see Willard S., General Topology [11], to study some concepts in topological space.

2 Preliminaries

Throughout, x denote topological spaces. Let A be a subset of X, then the closure and the interior of A are denoted by $Cl(A)$ and $Int(A)$ respectively. A subset A of a topological space (X,τ) is said to be semi open[12](resp. pre open[13], α-open[14], β-open[15]) if $A \subseteq Cl(Int(A))$ (resp. $A \subseteq Int(Cl(A))$, $A \subseteq Int(Cl(Int(A)))$, $A \subseteq Cl(Int(Cl(A)))$).

The family of all semi open (resp. pre open, α-open, β-open) sets in X is denoted by $SO(X,\tau)$ or $SO(X)$ (resp. $PO(X)$, $\alpha O(X)$, $\beta O(X)$). The complement of a semi open (resp. pre open, α-open, β-open) set is semi-closed (resp. pre closed , α-closed , β-closed). The family of all semi closed sets in a topological space (X,τ) is denoted by $SC(X,\tau)$ or $SC(X)$. We consider λ as a function defined on $SO(X)$ into $P(X)$ and $\lambda : SO(X) \to P(X)$ is called an s-operation if $V \subseteq \lambda(V)$ for each non-empty semi open set V. It is assumed that $\lambda(\phi) = \phi$ and $\lambda(X) = X$ for any s-operation λ.

Definition 2.1.[9].Let (X,τ) be a topological space and $\lambda : SO(X) \to P(X)$ be an s-operation, then a subset A of X is called a λ-open set or λ^*-open set if for each $x \in A$ there exists a semi open set U such that $x \in U$ and $\lambda(U) \subseteq A$. The complement of a λ^*-open set is said to be λ^*-closed. The family of all λ^*-open (resp., λ^*-closed) subsets of a topological space (X,τ) is denoted by $SO_\lambda(X,\tau)$ or $SO_\lambda(X)$ (resp. , $SC_\lambda(X,\tau)$ or $SC_\lambda(X)$).

Proposition 2.2.[7],[16].For a topological space (X,τ), $SO_\lambda(X) \subseteq SO(X)$.

The following examples show that the converse of the above proposition may not be true in general.

Example 2.3.[7],[16]. Let $X=\{a,b,c\}$, and $\tau=\{\phi,\{a\},X\}$. We define an s-operation $\lambda: SO(X)\rightarrow P(X)$ as $\lambda(A)=A$ if $b\in A$ and $\lambda(A)=X$ otherwise. Here, we have $\{a,c\}$ is semi open set but it is not λ^*-open.

Definition 2.4.[7],[16]. Let (X,τ) be a space, an s-operation λ is said to be s-regular if for every semi open sets U and V of $x\in X$, there exists a semi open set W containing x such that $\lambda(W)\subseteq\lambda(U)\cap\lambda(V)$.

Definition 2.5.[8]. Let (X,τ) be a topological space and let A be a subset of X. Then:

(1) The λ^*-closure of A ($\lambda Cl(A)=\lambda^* Cl(A)$) is the intersection of all λ^*-closed sets containing A.

(2) The λ^*-interior of A ($\lambda Int(A)=\lambda^* Int(A)$) is the union of all λ^*-open sets of X contained in A.

(3) A point $x\in X$, is said to be a λ^*-limit point of A if every λ^*-open set containing x contains a point of A different from x, and the set of all λ^*-limit points of A is called the λ^*-derived set of A denoted by $\lambda d(A)=\lambda^* d(A)$.

Proposition 2.6.[7],[16]. For each point $x\in X$, $x\in\lambda Cl(A)$ if and only if $V\cap A\neq\phi$, for every $V\in SO_\lambda(X)$ such that $x\in V$.

Proposition 2.7.[7],**[16]**

Let $\{A_\alpha\}_{\alpha\in I}$ be any collection of λ^*-open sets in a topological space (X,τ) then $\bigcup_{\alpha\in I} A_\alpha$ is a λ^*-open set.

The following example shows that the intersection of two λ^*-open sets need not be λ^*-open.

Example 2.8.[7],**[16]**

Let $X=\{a,b,c\}$, and $\tau=P(X)$. We define an s-operation $\lambda: SO(X)\to P(X)$ as:

$$\lambda(A)=\begin{cases} A & \text{if} \quad A\neq\{a\},\{b\} \\ X & \text{if} \quad A=\{a\} \text{ or } \{b\} \end{cases}.$$

We have $\{a,b\}$ and $\{b,c\}$ are λ^*-open sets but $\{a,b\}\cap\{b,c\}=\{b\}$ is not λ^*-open.

From Proposition 2.7 and the above example we notice that the family of all λ^*-open sets of a space X is a supra topology and need not be a topology in general.

Example 2.9.[7],**[16]**

Let $X=\{a,b,c\}$, and $\tau=P(X)$. We define an s-operation $\lambda: SO(X)\to P(X)$ as:

$$\lambda(A)=\begin{cases} A & \text{if } A=\{b\} \text{ or } \{a,b\} \text{ or } \{a,c\} \text{ or } \phi \\ X & \quad \text{Otherwise} \end{cases}.$$

Then we can easily find the following families of sets:

$SO(X)=P(X)=\{\phi,\{a\},\{b\},\{c\},\{a,b\},\{a,c\},\{b,c\},X\}$;

$SO_\lambda(X)=\{\phi,\{b\},\{a,b\},\{a,c\},X\}$;

Proposition 2.10. [7],**[16].** Let λ be an s-regular s-operation. If A and B are λ^*-open sets in X, then $A\cap B$ is also a λ^*-open set.

Proposition 2.11. [7],**[16]**. Let (X,τ) be a topological space and $A\subseteq X$. Then A is a λ^*-closed subset of X if and only if $\lambda d(A)\subseteq A$.

Proposition 2.12.[7],**[16].** For subsets A,B of a topological space (X,τ), the following statements are true.

(1) $A\subseteq\lambda Cl(A)$.

(2) $\lambda Cl(A)$ is λ^*-closed set in X.

(3) $\lambda Cl(A)$ is smallest λ^*-closed set which contain A.

(4) A is λ^*-closed set if and only if $A=\lambda Cl(A)$.

(5) $\lambda Cl(\phi)=\phi$ and $\lambda Cl(X)=X$.

(6) If A and B are subsets of space X with $A\subseteq B$. Then $\lambda Cl(A)\subseteq \lambda Cl(B)$.

(7) For any subsets A,B of a space (X,τ), $\lambda Cl(A)\cup \lambda Cl(B)\subseteq \lambda Cl(A\cup B)$.

(8) For any subsets A,B of a space (X,τ), $\lambda Cl(A\cap B)\subseteq \lambda Cl(A)\cap \lambda Cl(B)$.

Proposition 2.13.[7],**[16]**. Let (X,τ) be a topological space and $A\subseteq X$. Then $\lambda Cl(A)=A\cup \lambda d(A)$.

Proposition 2.14.[7],**[16]**. For a subset A of a topological space (X,τ), $\lambda Int(A)=A\setminus \lambda d(X\setminus A)$.

Proposition 2.15. [7],**[16]**. For any subset A of a topological space X. The following statements are true.

(1) $X\setminus \lambda Int(A)=\lambda Cl(X\setminus A)$.

(2) $\lambda Cl(A)=X\setminus \lambda Int(X\setminus A)$.

(3) $X\setminus \lambda Cl(A)=\lambda Int(X\setminus A)$.

(4) $\lambda Int(A)=X\setminus \lambda Cl(X\setminus A)$.

Theorem 2.16.[7],**[16]**. Let A,B be subsets of X. If $\lambda : SO(X)\rightarrow P(X)$ is an s-regular s-operation, then:

(1) $\lambda Cl(A\cup B)=\lambda Cl(A)\cup \lambda Cl(B)$.

(2) $\lambda Int(A\cap B)=\lambda Int(A)\cap \lambda Int(B)$.

(3)

We can define the following example and remark:

Definition 2.17

Let (X,τ) be a topological space and let $x\in X$. A subset N of X is said to be a λ^*-neighbourhood (λ^*-nhd) of x if and only if there exists a λ^*-open set V such that $x\in V\subseteq N$. The family of all λ^*-neighbourhood of x, denoted by $\lambda N(x)$.

Remark 2.18

Every λ^*-open set in X which contain $x \in X$ is λ^*-neighbourhood of x, but conversely is not true, in Example 2.7, we have $\{b,c\}$ is λ^*-nhd of b, but it is not λ^*-open set

Definition 2.19

Let (X,τ) and (Y,σ) be two topological spaces. A function $f:(X,\tau) \to (Y,\sigma)$ is called:

(1) Irresolute[17], if $f^{-1}(V)$ is semi open in X for every semi open set V of Y.

(2) Pre continuous[13], if $f^{-1}(V)$ is pre open in X for every open set V of Y.

(3) Semi continuous[12], if $f^{-1}(V)$ is a semi open set in X, for each open set V in Y.

(4) α-continuous[18], if $f^{-1}(V)$ is α-open in X for every open set V of Y.

(5) Contra-semi-continuous[4], if $f^{-1}(V)$ is semi closed in X for each open set V of Y.

(6) α-irresolute[19], if $f^{-1}(V)$ is α-open set in X for each α-open set V of Y.

(7) β-continuous[15], if $f^{-1}(V)$ is β-open in X for every open set V of Y.

(8) β-irresolute[20], if $f^{-1}(V)$ is β-open in X for every β-open set V of Y.

(9) Pre-irresolute[21], if $f^{-1}(V)$ is pre open set in X for each pre open set V of Y.

Proposition 2.20[9]

For any topological space (X,τ), we have:

(1) If $SO(X)$ is indiscrete, then $SO_\lambda(X)$ is also indiscrete.

(2) If $SO_\lambda(X)$ is discrete, then $SO(X)$ is also discrete.

Definition 2.21.[7]. A subset A of a topological space (X,τ) is said to be generalized λ-closed (briefly. g-λ-closed) if $\lambda Cl(A) \subseteq U$, whenever $A \subseteq U$ and U is a λ-open set in (X,τ).

We say that a subset B of X is generalized λ-open (briefly. g-λ-open) if its complement $X \setminus B$ is generalized λ-closed in (X,τ).

In the following proposition we show every λ-closed subset of X is g-λ-closed.

Proposition 2.22.[7]. Every λ-closed set is g-λ-closed.

The reverse claim in the above proposition is not true in general. Next we give an example of a g-λ-closed set which is not λ-closed.

Example 2.23.[7]. Let $X = \{a,b,c\}$, and $\tau = P(X)$. We define an s-operation $\lambda : SO(X) \rightarrow P(X)$ as $\lambda(A) = A$ if $A = \{a\}$ and $\lambda(A) = X$ otherwise. Then, if we let $A = \{a,b\}$, and since the only λ-open supersets of A is X, so A is g-λ-closed but it is not λ-closed.

Proposition 2.24.[7]. The intersection of a g-λ-closed set and a λ-closed set is always g-λ-closed.

We used[7],[16] for getting the following results.

We introduce the concept of $(\lambda,\gamma)^*$-continuous function and study some of its basic properties. Also we define $(\lambda,\gamma)^*$-open(closed) functions, moreover some properties of these functions are studied. Throughout, (X,τ), (Z,ρ) and (Y,σ) are topological spaces and λ,η and γ are s-operations on the family of semi open sets of the topological spaces respectively.

Definition 2.25

A function $f:(X,\tau) \rightarrow (Y,\sigma)$ is said to be $(\lambda,\gamma)^*$-continuous, if for each x of X and each γ^*-open set V of Y containing $f(x)$ there exists a λ^*-open set U of X such that $x \in U$ and $f(U) \subseteq V$.

Theorem 2.26

Let $f:(X,\tau)\to(Y,\sigma)$ be a function, then f is $(\lambda,\gamma)^*$-continuous if and only if for each γ^*-open set B in Y, $f^{-1}(B)$ is λ^*-open in X.

By the followings examples we can show that a $(\lambda,\gamma)^*$-continuous function is different from continuous (semi continuous, α-continuous, pre continuous, β-continuous, irresolute, α-irresolute, pre irresolute, β-irresolute) function in general.

Example 2.27

Let $X=Y=\{a,b,c\}$, $\tau=P(X)$ and $\sigma=P(Y)$. We define an s-operation $\lambda:SO(X)\to P(X)$ by :

$$\lambda(A)=\begin{cases} A & \text{if} \quad A=\{c\} \text{ or } \{a,b\} \text{ or } \{a,c\} \text{ or } \phi \\ X & \text{Otherwise} \end{cases}.$$

And $\gamma:SO(Y)\to P(Y)$ be a γ-identity s-operation. Then the identity function $f:(X,\tau)\to(Y,\sigma)$ is continuous, semi continuous, α-continuous, pre continuous, β-continuous, irresolute, α-irresolute, pre irresolute, β-irresolute, but it is not $(\lambda,\gamma)^*$-continuous since $\{b\}$ is γ^*-open set but $f^{-1}(\{b\})=\{b\}$ is not λ^*-open.

Example 2.28

Let $X=\{a,b,c\}$, and $\tau=\{\phi,\{a\},\{c\},\{a,c\},\{b,c\},X\}$. We define an s-operation $\lambda:SO(X)\to P(X)$ by:

$$\lambda(A)=\begin{cases} A & \text{if} \quad A=\{a\} \text{ or } \phi \\ X & \text{Otherwise} \end{cases}.$$

The function $f:(X,\tau)\to(X,\tau)$ defined by $f(b)=c$, $f(c)=b$ and $f(a)=a$ is $(\lambda,\gamma)^*$-continuous, but it is not continuous, semi continuous, α-continuous, pre continuous, β-continuous, irresolute, α-irresolute, pre irresolute and β-irresolute. Since $\{c\}$ is open set and $f^{-1}(\{c\})=\{b\}$, but $\{b\}$ is not open, semi open, α-open, pre-open and β-open.

Proposition 2.29

(1) If $f:(X,\tau)\rightarrow(Y,\sigma)$ is $(\lambda,\gamma)^*$-continuous and (Y,σ) is indiscrete space, then f is semi continuous.

(2) If $SO_\lambda(X)$ is discrete space, then any function $f:(X,\tau)\rightarrow(Y,\sigma)$ is $(\lambda,\gamma)^*$-continuous.

Theorem 2.30

Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be a function. Then the following statements are equivalent:

(1) f is $(\lambda,\gamma)^*$-continuous.

(2) The inverse image of each γ^*-closed set in *Y* is a λ^*-closed set in *X*.

(3) $\lambda Cl(f^{-1}(V))\subseteq f^{-1}(\gamma Cl(V))$, for every $V\subseteq Y$.

(4) $f(\lambda Cl(U))\subseteq \gamma Cl(f(U))$, for every $U\subseteq X$;

(5) $\lambda Bd(f^{-1}(V))\subseteq f^{-1}(\gamma Bd(V))$, for every $V\subseteq Y$.

(6) $f(\lambda d(U))\subseteq \gamma Cl(f(U))$, for every $U\subseteq X$.

(7) $f^{-1}(\gamma Int(V))\subseteq \lambda Int(f^{-1}(V))$, for every $V\subseteq Y$.

Proposition 2.31

If the functions $f:(X,\tau)\rightarrow(Z,\rho)$ is $(\lambda,\eta)^*$-continuous and $g:(Z,\rho)\rightarrow(Y,\sigma)$ is $(\eta,\gamma)^*$-continuous, then $g\circ f:(X,\tau)\rightarrow(Y,\sigma)$ is $(\lambda,\gamma)^*$-continuous.

Definition 2.32

A function $f:(X,\tau)\rightarrow(Y,\sigma)$ is said to be $(\lambda,\gamma)^*$-open ($(\lambda,\gamma)^*$-closed), if for any λ^*-open (λ^*-closed) set *A* of (X,τ), $f(A)$ is γ^*-open (γ^*-closed).

Theorem 2.33

Suppose that $f:(X,\tau)\rightarrow(Y,\sigma)$ is $(\lambda,\gamma)^*$-continuous and $(\lambda,\gamma)^*$-closed function, then:

(1) For every g-λ^*-closed set A of (X,τ) the image $f(A)$ is a g-γ^*-closed set.

(2) For every g-γ^*-closed set B of (Y,σ) the inverse set $f^{-1}(B)$ is a g-λ^*-closed set.

Corollary 2.34

If $f:(X,\tau)\to(Y,\sigma)$ is a bijective function, then the following statement are equivalent.

(1) f is $(\lambda,\gamma)^*$-homeomorphism.

(2) $f(\lambda Cl(A)) = \gamma Cl(f(A))$ for all $A \subseteq X$.

(3) $\lambda Cl(f^{-1}(B)) = f^{-1}(\gamma Cl(B))$ for all $B \subseteq Y$.

(4) $f(\lambda Int(A)) = \gamma Int(f(A))$ for all $A \subseteq X$.

(5) $\lambda Int(f^{-1}(B)) = f^{-1}(\gamma Int(B))$ for all $B \subseteq Y$.

3.1 Contra (λ,γ)-Continuous Function

In this section, we introduce the concept of contra $(\lambda,\gamma)^*$-continuous function and study some of its basic properties, also we compare it with $(\lambda,\gamma)^*$-continuous, and other types of functions. Moreover, we give a new property of functions which we call $(\lambda,\gamma)^*$-interior property.

Definition 3.1

A function $f:(X,\tau)\to(Y,\sigma)$, is said to be contra $(\lambda,\gamma)^*$-continuous if for every γ^*-open subset H of Y, $f^{-1}(H)$ is λ^*-closed in X.

Definition 3.2

For any s-operation $\lambda : SO(X) \rightarrow P(X)$ and any subset A of a space (X,τ), the λ^*-kernel of A, denoted by $\lambda Ker(A)$, is defined as:

$\lambda Ker(A)$ ($\lambda^* Ker(A)$ [8])= $\bigcap$ { $G \in SO_\lambda(X) : A \subseteq G$ }.

Lemma 3.3

Let X be a space, and $\lambda : SO(X) \rightarrow P(X)$ be an s-operation and $A \subseteq X$. Then $\lambda Ker(A)$ ={ $x \in X : \lambda Cl(\{x\}) \cap A \neq \phi$}.

Proof: Let $x \in \lambda Ker(A)$ and $\lambda Cl(\{x\}) \cap A = \phi$. Then $x \notin X \setminus \lambda Cl(\{x\})$, which is a λ^*-open set containing A. Thus $x \notin \lambda Ker(A)$, a contradiction.

Conversely, let $x \in X$ be such that $\lambda Cl(\{x\}) \cap A \neq \phi$. If possible, let $x \notin \lambda Ker(A)$. Then there exist a λ^*-open set G such that $x \notin G$ and $A \subseteq G$. Let $y \in \lambda Cl(\{x\}) \cap A$. This implies that $y \in \lambda Cl(\{x\})$ and $y \in G$, which gives $x \in G$, a contradiction.

Theorem 3.4

Let (X,τ) be a topological space, A and B be a subsets of X. Then:

(1) $x \in \lambda Ker(A)$ if and only if $A \cap F \neq \phi$; for any λ^*-closed set F that contains x.

(2) $A \subseteq \lambda Ker(A)$ and $A = \lambda Ker(A)$ if A is λ^*-open.

(3) If $A \subseteq B$, then $\lambda Ker(A) \subseteq \lambda Ker(B)$.

Proof. Obvious.

Theorem 3.5

For a function $f : (X,\tau) \rightarrow (Y,\sigma)$, the following properties are equivalent

(1) f is contra $(\lambda,\gamma)^*$-continuous.

(2) for every γ^*-closed subset F of Y, $f^{-1}(F)$ is λ^*-open in X.

(3) for each $x \in X$ and each γ^*-closed subset F of Y containing $f(x)$, there exists a λ^*-open set U of X containing x such that $f(U) \subseteq F$.

(4) $f(\lambda Cl(A)) \subseteq \gamma Ker(f(A))$ for every subset A of X.

(5) $\lambda Cl(f^{-1}(B)) \subseteq f^{-1}(\gamma Ker(B))$ for every subset B of Y.

Proof. The equivalences of (1) and (2) and (3) are obvious.

$(\mathbf{2}) \Rightarrow (\mathbf{4})$: Let A be any subset of X. Suppose that $y \notin \gamma Ker(f(A))$. Then by Lemma 3.3, there exists a γ^*-closed set F containing y such that $f(A) \cap F = \phi$ .Thus, we have $A \cap f^{-1}(F) = \phi$ and since $f^{-1}(F)$ is λ^*-open we have $\lambda Cl(A) \cap f^{-1}(F) = \phi$. Therefore, we obtain $f(\lambda Cl(A)) \cap F = \phi$ hence $y \notin f(\lambda Cl(A))$. This implies that $f(\lambda Cl(A)) \subseteq \gamma Ker(f(A))$.

$(\mathbf{4}) \Rightarrow (\mathbf{5})$: Let B be any subset of Y. By (4), we have $f(\lambda Cl(f^{-1}(B))) \subseteq \gamma Ker(f(f^{-1}(B))) \subseteq \gamma Ker(B)$ and thus $\lambda Cl(f^{-1}(B)) \subseteq f^{-1}(\gamma Ker(B))$.

$(\mathbf{5}) \Rightarrow (\mathbf{1})$: Let V be any γ^*-open set of Y. Then, we have $\lambda Cl(f^{-1}(V)) \subseteq f^{-1}(\gamma Ker(V)) = f^{-1}(V)$ and $\lambda Cl(f^{-1}(V)) = f^{-1}(V)$. This shows that $f^{-1}(V)$ is λ^*-closed in X.

Remark 3.6

In fact contra $(\lambda,\gamma)^*$-continuity and $(\lambda,\gamma)^*$-continuity are independent .

Example 3.7

Let $X=\{a,b\}=Y$, $\tau=P(X)$ and $\sigma=P(Y)$. We define an s-operation

$\lambda : SO(X) \to P(X)$ as:

$\lambda(A)=\begin{cases} A & \text{if } A=\{a\} \text{ or } \phi \\ X & \text{Otherwise} \end{cases}$. Also the s-operation $\gamma: SO(Y)\rightarrow P(Y)$ defined as:

$\gamma(B)=\begin{cases} B & \text{if } B=\{b\} \text{ or } \phi \\ Y & \text{Otherwise} \end{cases}$. Then the identity function $f:(X,\tau)\rightarrow(X,\sigma)$ is contra $(\lambda,\gamma)^*$-continuous but it is not $(\lambda,\gamma)^*$-continuous, since we have $\{b\}$is λ^*-closed set but $f^{-1}(\{b\})=\{b\}\notin SO_\lambda(X)$.

Example 3.8

Let $X=\{a,b\}=Y$, $\tau=P(X)$ and $\sigma=P(Y)$. We define an s-operation $\lambda: SO(X)\rightarrow P(X)$ by :

$\lambda(A)=\begin{cases} A & \text{if } A=\{a\} \text{ or } \phi \\ X & \text{Otherwise} \end{cases}$. Also the s-operation $\gamma: SO(Y)\rightarrow P(Y)$ defined as:

$\gamma(B)=\begin{cases} B & \text{if } B=\{b\} \text{ or } \phi \\ Y & \text{Otherwise} \end{cases}$. A function $f:(X,\tau)\rightarrow(X,\sigma)$ defined as $f(a)=b$ and $f(b)=a$ is $(\lambda,\gamma)^*$-continuous but it is not contra $(\lambda,\gamma)^*$-continuous, since we have $\{a\}$ is λ^*-closed set but $f^{-1}(\{a\})=\{b\}\notin SO_\lambda(X)$.

Definition 3.9

A function $f:(X,\tau)\rightarrow(Y,\sigma)$ is said to satisfy the $(\lambda,\gamma)^*$-interiority condition if $\lambda Int(f^{-1}(\gamma Cl(V)))\subseteq f^{-1}(V)$ for each γ^*-open set V of (Y,σ).

Theorem 3.10

If $f:(X,\tau)\rightarrow(Y,\sigma)$ is a contra $(\lambda,\gamma)^*$-continuous function and satisfies the $(\lambda,\gamma)^*$-interiority condition, then f is $(\lambda,\gamma)^*$-continuous.

Proof. Let V be any γ^*-open subset of Y. Since f is contra $(\lambda,\gamma)^*$-continuous and $\gamma Cl(V)$ is γ^*-closed, by Theorem 3.5, $f^{-1}(\gamma Cl(V))$ is λ^*-open in X. By the hypothesis on f, $f^{-1}(V) \subseteq f^{-1}(\gamma Cl(V)) = \lambda Int(f^{-1}(\gamma Cl(V))) \subseteq \lambda Int(f^{-1}(V)) \subseteq f^{-1}(V)$. Therefore, we obtain $\lambda Int(f^{-1}(V)) = f^{-1}(V)$ and consequently $f^{-1}(V)$ is λ^*-open set in X. This shows that f is a $(\lambda,\gamma)^*$-continuous function.

Through the following examples we can show that contra $(\lambda,\gamma)^*$-continuous and contra-semi-continuity are independent concepts :

Example 3.11

Let $Y = X = \{a,b,c\}$, and $\tau = P(X)$. We define an s-operation $\lambda : SO(X) \rightarrow P(X)$ as:

$\lambda(A) = \begin{cases} A & \text{if } A = \{a\} \text{ or } \{c\} \text{ or } \phi \\ X & \text{Otherwise} \end{cases}$. If $\sigma = P(Y)$ and γ is γ-identity s-operation, then the function $f:(X,\tau) \rightarrow (Y,\sigma)$, defined by $f(a)=c$ and $f(b)= f(c)=b$ is contra-semi-continuous, but it is not contra $(\lambda,\gamma)^*$-continuous.

Example 3.12

Let $X = \{a,b\} = Y$, with spaces $\tau = \{\phi, X\}$ and $\sigma = \{\phi,\{b\},Y\}$. We define an s-operation $\lambda : SO(X) \rightarrow P(X)$ as $\lambda(A) = X$ for all $\phi \neq A \subseteq X$ and an s-operation $\gamma : SO(Y) \rightarrow P(Y)$ as $\gamma(B) = Y$ for all $\phi \neq B \subseteq X$. Then the identity function $f:(X,\tau) \rightarrow (X,\sigma)$ is contra $(\lambda,\gamma)^*$-continuous but it is not contra-semi-continuous.

Definition 3.13

A topological space (X,τ) is said to be locally λ^*-indiscrete if every λ^*-open set of X is λ^*-closed in *X*.

Through the following examples we can show that the property locally λ^*-indiscrete and locally indiscrete are independent.

Example 3.14

Let $X=\{a,b,c\}$, and $\tau=P(X)$. We define an s-operation $\lambda:SO(X)\to P(X)$ as:

$\lambda(A)=\begin{cases} A & \text{if } A=\{a\} \text{ or } \{c\} \text{ or } \phi \\ X & \text{Otherwise} \end{cases}$. Clearly (X,τ) is locally indiscrete, but it is not locally λ^*-indiscrete, because $\{a\}\in SO_\lambda(X)$ but $\{a\}\notin SC_\lambda(X)$.

Example 3.15

Let $X=\{a,b,c\}$, and $\tau=\{\phi,\{a\},\{b\}\{a,b\},X\}$. Define an s-operation $\lambda:SO(X)\to P(X)$ as $\lambda(A)=X$ for all $\phi\neq A\subseteq X$. Clearly (X,τ) is locally λ^*-indiscrete, but it is not locally indiscrete, because $\{a\}$ is an open set but it is not closed.

Theorem 3.16

If $f:(X,\tau)\to(Y,\sigma)$ is a function and *X* is locally λ^*-indiscrete, then f is $(\lambda,\gamma)^*$-continuous if and only if f is contra $(\lambda,\gamma)^*$-continuous.

Proof. Let *H* be any γ^*-open set in *Y*. Then by hypothesis $f^{-1}(H)$ λ^*-open set in *X*, from the Definition 3.13, $f^{-1}(H)$ λ^*-closed set in *X*. Hence *f* is contra $(\lambda,\gamma)^*$-continuous.

Conversely, Let B be any γ^*-closed set in Y. Then $f^{-1}(B)$ is a λ^*-open set in X by Theorem 3.5(2). Since X is locally λ^*-indiscrete, so $f^{-1}(B)$ is a λ^*-closed set in X. Hence f is $(\lambda,\gamma)^*$-continuous by Theorem 2.30(2).

The composition of two contra $(\lambda,\gamma)^*$-continuous functions need not be contra $(\lambda,\gamma)^*$-continuous.

Example 3.17

Let $X=\{a,b\}=Y$, $\tau=P(X)$ and $\sigma=P(Y)$. We define an s-operation $\lambda: SO(X)\to P(X)$ such that:

$\lambda(A)=\begin{cases} A & \text{if } A=\{a\} \text{ or } \phi \\ X & \text{Otherwise}\end{cases}$. Also the s-operation $\gamma: SO(Y)\to P(Y)$ defined as:

$\gamma(B)=\begin{cases} B & \text{if } B=\{b\} \text{ or } \phi \\ Y & \text{Otherwise}\end{cases}$.

Then the identity functions $f:(X,\tau)\to(X,\sigma)$ and $g:(X,\sigma)\to(X,\tau)$ are contra $(\lambda,\gamma)^*$-continuous but $g\circ f:(X,\tau)\to(X,\tau)$ is not contra $(\lambda,\gamma)^*$-continuous.

Theorem 3.18

Let $f:(X,\tau)\to(Z,\rho)$ and $g:(Z,\rho)\to(Y,\sigma)$ be two functions. Then:

(1) $g\circ f:(X,\tau)\to(Y,\sigma)$ is contra $(\lambda,\gamma)^*$-continuous, if g is $(\eta,\gamma)^*$-continuous and f is contra $(\lambda,\eta)^*$-continuous.

(2) $g\circ f$ is contra $(\lambda,\gamma)^*$-continuous, if g is contra $(\eta,\gamma)^*$-continuous and f is $(\lambda,\eta)^*$-continuous.

(3) $g\circ f$ is contra$(\lambda,\gamma)^*$-continuous, if g and f are $(\eta,\gamma)^*$-continuous and $(\lambda,\eta)^*$-continuous respectively and (Z,ρ) is locally η^*-indiscrete.

Proof. (**1**) Let $V\in SO_\gamma(Y)$. Then $g^{-1}(V)\in SO_\eta(Z)$ and $f^{-1}(g^{-1}(V))\in SO_\lambda(X)$ since g is $(\eta,\gamma)^*$-continuous and f is contra$(\lambda,\eta)^*$-continuous. It follows that $(g\circ f)^{-1}\in SC_\lambda(X)$. Hence $g\circ f$ is contra$(\lambda,\gamma)^*$-continuous.

(**2**) Let $V\in SO_\gamma(Y)$, then $g^{-1}(V)\in SC_\eta(Z)$ and $f^{-1}(g^{-1}(V))\in SC_\lambda(X)$ since g is contra $(\eta,\gamma)^*$-continuous and f is $(\lambda,\eta)^*$-continuous. It follows that $(g\circ f)^{-1}\in SC_\lambda(X)$. Hence $g\circ f$ is contra$(\lambda,\gamma)^*$-continuous.

(**3**) Let $V\in SO_\gamma(Y)$, then $g^{-1}(V)\in SO_\eta(Z)$ and $g^{-1}(V)\in SC_\eta(Z)$ since g is $(\eta,\gamma)^*$-continuous and (Z,ρ) is locally η^*-indiscrete, then $f^{-1}(g^{-1}(V))\in SO_\lambda(X)$ since f is $(\lambda,\eta)^*$-continuous. Hence $g\circ f$ is contra$(\lambda,\gamma)^*$-continuous.

References

1. Kasahara, S., *Operation-compact spaces*. Math. Japon., 1979. **24**: p. 97-105.
2. Ogata, H., *Operations on topological spaces and associated topology*. Math. Japon., 1991. **36**: p. 175-184.
3. Rehman, F. and B. Ahmad, *Operations on topological spaces I*. Math. Today, 1992. **10**: p. 29-36.
4. Jankovic, D., *On functions with α-closed graphs*. Glas. Mat., 1983. **18**(38): p. 141-148.
5. B. Ahmad, S.H., *Properties of γ-operations in topological spaces*. Aligarh Bull. Math, 2003: p. 45-51.
6. S., H., *Gamma-Operations in Topological Spaces*. Bahauddin Zakariya University Multan, 2007.
7. Alias B. Khalaf, S.F.N., *Generalized λ-Closed Sets and 〚(λ,γ)〛^*-Continuous Functions*. International Journal of Scientific & Engineering Research, 2012. **3**(12).
8. F.Namiq, S., *λ^*-R_0 and λ^*-R_1 Spaces*. Journal of Garmyan University, 2014(3).

9. F.Namiq, S., *New types of continuity and separation axiom based operation in topological spaces*. 2011, Sulaimani.
10. Khalaf, A.B. and S.F. Namiq, *[[lambda]. sub. c]-open sets and [[lambda]. sub. c]-separation axioms in topological spaces.* Journal of Advanced Studies in Topology, 2013. **4**(1): p. 150-159.
11. Willard, S., *General topology*. 1970: Courier Corporation.
12. Levine, N., *Semi-open sets and semi-continuity in topological spaces.* The American Mathematical Monthly, 1963. **70**(1): p. 36-41.
13. Mashhour, A. *On precontinuous and weak precontinuous mappings*. in *Proc. Math. Phys. Soc. Egypt*. 1982.
14. Njåstad, O., *On some classes of nearly open sets.* Pacific journal of mathematics, 1965. **15**(3): p. 961-970.
15. El-Monsef, M.A., S. El-Deeb, and R. Mahmoud, *β-open sets and β-continuous mappings.* Bull. Fac. Sci. Assiut Univ, 1983. **12**(1): p. 77-90.
16. Alias B. Khalaf, S.F.N., *Generalized λ-Closed Sets and 〚(λ,γ)〛^*-Continuous Functions.* Journal of Garmian University 2017. **Preprint**.
17. Crossley, S. and S. Hildebrand, *Semi-topological properties.* Fundamenta Mathematicae, 1972. **74**(3): p. 233-254.
18. Mashhour, A., I. Hasanein, and S. El-Deeb, *α-continuous and α-open mappings.* Acta Mathematica Hungarica, 1983. **41**(3-4): p. 213-218.
19. Maheshwari, S. and S. Thakur, *On α-irresolute mappings.* Tamkang J. Math, 1980. **11**(2): p. 209-214.
20. Mahmoud, R. and M.A. El-Monsef, *β-irresolute and β-topological invariant.* Proc. Pakistan Acad. Sci, 1990. **27**(3): p. 285-296.
21. K, R.I.L.a.V.M., *On α -continuity in topological spaces.* Acta Mathematica Hungarica 1985. **45**((1-2)): p. 27-32.